REFEREE REPORT FOR SEMR: "ON THE EXIXSTENCE OF NEF-PARTITIONS FOR SMOOTH WELL-FORMED FANO WCI'S" BY M.A. OVCHARENKO

Nef partitions are interesting combinatorial structures on smooth Fano weighted complete intersections, also useful to understand the Mirror symmetry of such manifolds. The main result of the note under review (Theorem 1.10) is a sufficient condition for the existence of strong nef paritions. The imposed condition (called "strongly non-divisible") looks quite strong and it would be nice to understand how strong it is. Note that strong nef partitions not always exist (see example 1.7).

The main technical result of the paper is Proposition 3.12, in which the author shows that any WCI induces a map between two abstract simplicial complexes constructed from the numerical data of the WCI. I think this is an interesting point of view.

The manuscript is well written and clear in the exposition and the proofs. I recommend it for publication.

I only have the following small remarks.

(1) My understanding is that Conjecture 1.2 is still open, is this correct?

(2) Typo: in Definition 2.3, "for any $i = 0, \ldots, N$".

(3) Typo: Last line of Lemma 2.6: no need of "the" in front of "index a_I".

(4) Typo: last line of page 5, "condition" should be plural.

(5) In Section 3, it could be useful to recall the definition of "abstract simplicial complex".

(6) Typo: Example 3.3 "an a set".

(7) Definition 3.11 and Proposition 3.12 talk about "face posets" and map between them. Is there a difference if one instead talks about map of abstract simplicial complexes?

(8) Proof of Proposition 3.12: I think it would be better to clarify a bit the proof. In particular, say explicitly why the conditions (1), (2) and (3) in the statement of Proposition 3.12 are satisfied by the construction. Also, the last part where a modification of $\chi^{(b_j)}$ is performed needs to be explained better.

(9) Typo: proof of Proposition 3.17, second line, "that" is unnecessary repeated twice.