Referee Report on the paper entitled

"Eigenfunction expansions of impulsive q-Sturm–Liouville problems "

written by Bilender P. Allahverdiev, Hüseyin Tuna and Hamlet A. Isayev.

The manuscript under review is devoted to the investigation of some spectral properties of the impulsive $q-$Sturm-Liouville problem defined by

$$-\frac{1}{q}D_{q^{-1}}D_q y\left(\zeta\right) + v\left(\zeta\right)y\left(\zeta\right) = \lambda y\left(\zeta\right), \ \zeta \in (0,d)\cup(d,a) \qquad (1)$$

$$y\left(0\right) - h_1 D_{q^{-1}}y\left(0\right) = 0,$$
$$y\left(d-\right) = \eta y\left(d+\right),$$
$$\mathcal{D}_{q^{-1}}y\left(d-\right) = \frac{1}{\eta}\mathcal{D}_{q^{-1}}y\left(d+\right),$$
$$y\left(a\right) + h_2 D_{q^{-1}}y\left(a\right) = 0, \qquad (2)$$

where $h_1, h_2, \eta > 0$, λ is a complex eigenvalue parameter, v is a real-valued continuous functions on $[0,d)\cup(d,q^{-1}a]$ and has finite limits $v(d\pm)$, $v\left(\zeta\right) \geq 0$, $\zeta \in [0,d)\cup(d,q^{-1}a]$, $a > 0$.

The manuscript consists of three sections. Section 1(Introduction) provides a summary of the relevant literature. In Section 2, the basic concepts of $q-$calculus are given. Section 3 contains the main results of the manuscripts. The authors proved some fundamental properties of Problem (1)-(2). First of all, it has been proven that the operator generated by Problem (1)-(2) is positive self-adjoint. The authors obtained the inverse of this operator. The compactness of the inverse operator has been achieved. Later, the authors obtained the existence of a countably infinite set of eigenvalues and eigenfunctions of Problem (1)-(2). Finally, using Steklov's method, they gave a uniformly convergent expansion formula in the eigenfunctions.

I think the idea of applying quantum calculus to the investigation of the impulsive Sturm-Liouville problems looks natural and the results could be of interest.

Conclusion: The theme of the manuscript is actual and corresponds to the profile of your journal. The results derived are original, mathematically correct, and contribute to the theory of differential operators.

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