

Referee report on paper
LOWER LARGE DEVIATIONS OF STRONGLY SUPERCRITICAL BRANCHING
PROCESS IN RANDOM ENVIRONMENT WITH GEOMETRIC NUMBER OF
DESCENDANTS: LOCAL ASYMPTOTICS

Results in the paper appear to be new and correct. I recommend publication but have a few comments that I hope could improve presentation:

- I think the reader would benefit from an explanation of what the random walk S_n represents. At the moment it is defined, without any comment, near the beginning of the paper and utilised extensively in the analysis.
- The paper studies what the author refers to as a phase transition in large deviations probabilities between two zones. What I feel the paper is missing is a discussion of (known from previous papers) results in each of the two zones and how results from this paper relate to them. It would benefit the paper if the phase transition was described and not merely mentioned.
- The last paragraph of Section 2 would be more useful if it became the first paragraph of Section 3. It would be more natural if a part of the main result was discussed right before its statement, rather than in a previous section.
- Theorem 2, the main result, is somewhat chaotic. The reader is reminded what $R(h)$ is, but not what is $m(-1)$; some further notation (e.g. μ) is repeated, while other (e.g. $\sigma(h)$) is not; the definition of k looks quite strange too. I would suggest first setting a scene before the theorem by referring the reader to some notation from the previous two sections, and then formulating a much shorter theorem.