

## Answer to referee's report

First of all, the authors would like to thank the referee for a thorough and deep reading of the manuscript and the substantial remarks. We took into account all referee's suggestions and give the detailed answers below.

1-3. - Corrected, thank you

4, also 40. of norm of flow  $\rightarrow$  of the norm of a flow.

- We corrected articles before flow and norm throughout the paper

5-7. - Corrected, thank you.

8. Inconsistence of incidence matrix dimensions.

- Thank you for this remark. Indeed, there is an inconsistence in matrix dimensions throughout Pages 6 and 7. We fixed it in Theorem 2, Example 1 and the proof of Theorem 3.3

9, 13, 14, 17, 23, 24, 26. Section 3.1, Statement of Theorem 3, Sentence 1. I recommend to introduce the set  $U$  before Theorem 3. I recommend to write  $U(n, q)$  instead of  $U$  (and  $U(n)$  for  $q = 1$ ). I also recommend to use  $U(n)$  in Section 4.

- We introduced  $U(n, q)$  prior to Theorem 3. Several passages using " $\sum_{i=1}^n u_i = 0$ " were changed to " $u \in U(n)$ " as you suggested in remarks 13,14,17,23,24,26.

10, 16, 28, 30, 36. Section 3.1, Statement of Theorem 3, Case 4. to the blocks of  $S$  is  $\theta_1(\Gamma_S)$ -eigenvector  $\rightarrow$  to the blocks of  $S$  is a  $\theta_1(\Gamma_S)$ -eigenvector.

- We corrected the articles before eigenvector here and in several places in the text.

11, 12. - Corrected, thank you.

15. Section 4.1, The proof of Lemma 1, penultimate sentence. divis or  $\rightarrow$  divisor

- Corrected, thank you. We also added the articles before the word "divisor" throughout the text.

18-20. Corrected, thank you.

21. Section 4.1, Statement of Lemma 2, Sentence 1.  $(n, s) = 1$   $(r, s) = 1$ ; and  $s$  is divisor of  $(n, k)$ ,. delete comma In view of Remark 9, I recommend to change  $\sum_{i=1}^n u_i = 0$  to "Let  $u$  be a vector from  $U(n)$ ".

- We have rewritten the Statement of Lemma 2 accordingly, thank you.

22. If  $W^T u$  is a NZI  $\theta_1(J(n, k))$ -eigenvector of  $J(n, k)$ ". I suggest to change it to "If  $W^T u$  is a NZI vector"

- It does look simpler without loss of context. Corrected, thank you.

- 25,29. Section 4.1, The proof of Corollary 1, Sentence 3. In the other hand  
→ On the other hand  
- We corrected this mistake in several places in the text.
27. Section 4.1, The proof of Corollary 2, Sentence 2. by Proposition 6 →  
due to Proposition 6  
- Corrected.
31. Section 4.1, The proof of Theorem 5, Sentence 3. In what follows n is  
odd → In what follows we assume that n is odd  
- Corrected.
- 32, also 37. Section 4.2, The proof of Theorem 6, Paragraph 1, Sentence 4.  
 $\theta_1(J(n, 3))$ -eigenvector →  $\theta_1(\Gamma_S)$ -eigenvector.  
- We fixed this mistake in several places in the text.
33. Section 4.2, The proof of Theorem 6, Paragraph 1, Sentence 6.  
 $(W_S^T)_{i,j,k} \rightarrow (W_S^T u)_{\{i,j,k\}}$   
- Corrected, thank you.
- 34, 38. Please explain lower bounds on  $\|W_S^T u\|_\infty$ .  
- The equality for norm is changed to upper bound.
35. Section 4.2, The proof of Theorem 6, Paragraph 2, Sentence 4. I suggest  
to delete "STS".
39. Section 5, Sentence 1. eigenvectors → eigenvectors.  
- Both suggestions were taken into account, thank you.
41. Section 5.1, The proof of Proposition 9, Paragraph 2, Sentence 1. "When  
r is odd, then it is easy to see that  $2^r$  is 2 modulo 6 by induction on r". I suggest  
to delete "by induction on r".
42. Section 5.1, The proof of Proposition 9, Paragraph 4, Sentence 1. Steiner  
triple systems constructed by Bose method of order 9p were re- cently shown to  
be resolvable → Steiner triple systems of order 9p con- structed by Bose method  
were recently shown to be resolvable  
- Both sentences are rewritten.
43. Section 5.2, Paragraph 1, the definition of S. It seems that  $P'(\{i, j, k\})$   
and  $P(\{i, j, k\})$  for the case  $\tau(\{i, j, k\}) = 1$  in the definition of S are inconsistent.  
- Thank you for noting this imporant mistake. The representation of  
Assmuss-Mattson construction via  $P(\{i, j, k\})$  is rewritten.
44. Section 5.2, Paragraph 2. "Our goal is to construct a zero-sum 5- ow in  
S". What is S'?  
The sentence is rewritten.
45. Section 5.2, The proof of Theorem 7, Sentence 1, from STS S → from a  
STS S  
- Rewritten, thank you.
- We also fixed other mistakes.** We list the most important changes.  
- We improved articles throughout the text.  
- Page 6, Statement of Theorem 2.2, Proposition 4.1 a vector  $u \rightarrow$  a nonzero  
vector  $u$ .  
- Page 6, Example 1, last sentence.  $\begin{bmatrix} 3 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 \\ 1 \end{bmatrix}_q$ . We fixed this mistake in  
several places in the text.  
- Page 7, Proof of Theorem 5, reduction → restriction

- Page 8 (now Page 9), the proof of Proposition 7, the third sentence. Divide the first  $n - (\beta + k + 1)$  positions into  $\gamma$  blocks  $\rightarrow$  Divide the first  $n - (\beta + k + 1)$  positions into  $q$  blocks of the same size  $\gamma$ .
- Page 9, the proof of Proposition 7, the end of penultimate paragraph: then this sum is equal to 0  $\rightarrow$  this equality does not hold.
- Extended the proof of Lemma 3.
- Page 11, the proof of Corollary 1, the first and the second sentences:  $(b - \frac{r}{s}) \rightarrow (-b + \frac{r}{s})$
- Page 11 (now Page 12), the proof of Corollary 2, the second sentence  $\|W^T u\|_\infty \rightarrow m(1, J(n, k))$
- Page 12, we rewrote the proof of Proposition 8. We also added Corollary 3 (which we see as having importance by itself) after Corollary 2.
- We gave more details in Theorem 7 explaining formula (8) (which is now formula (10))