Referee's report on

L_∞ norm minimization for nowhere-zero integer eigenvectors of the block graphs of Steiner triple systems and Johnson graphs

In the paper under review, nowhere-zero integer eigenvectors of the Johnson graphs and the block graphs of Steiner triple systems are studied. In Section 4.1, the authors establish lower and upper bounds for the minimum value of $\|v\|_\infty$, where v is a nowhere-zero integer eigenvector of the Johnson graph associated with its first eigenvalue. In Section 4.2, the authors prove upper bounds for the minimum value of $\|v\|_\infty$, where v is a nowhere-zero integer eigenvector of the block graph of a Steiner triple system associated with its first eigenvalue. In Section 5, the authors show that any Assmuss-Mattson Steiner triple system obtained from a Steiner triple system of order at least 49 has a 5-flow. Finally, in Section 6, the authors study completely regular codes in the block graphs of Steiner triple systems.

The main results of the paper are interesting and actual. I recommend publishing the paper after revision.

Remarks:

1. Introduction, Paragraph 2, Sentence 1.
   of Steiner triple system $\longrightarrow$ of a Steiner triple system
2. Introduction, Paragraph 2, Sentence 2.
   vector u: $W_S u = 0 \longrightarrow$ vector u such that $W_S u = 0$
3. Introduction, Paragraph 3, Sentence 2.
   in Steiner triple system $\longrightarrow$ in a Steiner triple system
4. Introduction, Paragraph 4, Sentence 1.
   of norm of flow $\longrightarrow$ of the norm of a flow
5. Introduction, Paragraph 5, Sentence 4.
   vector v: $Wv = 0 \longrightarrow$ vector v such that $Wv = 0$
6. Section 2.1, Paragraph 1, the last sentence.
   , note that $\longrightarrow$. Note that
7. Section 2.1, The penultimate paragraph, Sentence 2.
   of q-ary, $q \geq 1$, STS S denoted by $\Gamma_S \longrightarrow$ of a q-ary, $q \geq 1$, STS S, denoted by Γ_S
8. Section 3, Statement of Theorem 2.
   If I is the incidence matrix of $V(\Gamma)$ and $V(\Gamma')$, then I is the $|V(\Gamma)|\times|V(\Gamma')|$ matrix. So, we cannot consider the product of I and u. Please, define the matrix I more carefully.

9. Section 3.1, Statement of Theorem 3, Sentence 1.

   I recommend to introduce the set U before Theorem 3. I recommend to write $U(n, q)$ instead of U (and $U(n)$ for $q = 1$). I also recommend to use $U(n)$ in Section 4.

10. Section 3.1, Statement of Theorem 3, Case 4.

    to the blocks of S is $\theta_1(\Gamma_S)$-eigenvector $\longrightarrow$ to the blocks of S is a $\theta_1(\Gamma_S)$-eigenvector

    each $\theta_1(\Gamma_S)$-eigenvectors $\longrightarrow$ each $\theta_1(\Gamma_S)$-eigenvector

11. Section 3.1, Paragraph 1, Sentence 2.

    is obtained deleting $\longrightarrow$ is obtained by deleting

12. In Section 4, you use formulas "$\sum_{i=1,\ldots,n} u_i$" many times. I suggest to change these formulas to "$\sum_{i=1}^{n} u_i$" (in all cases).

13. Section 4.1, Statement of Lemma 1, Sentence 1.

    "where u is a real-valued eigenvector of J(n,1), $\sum_{i=1,\ldots,n} u_i = 0$". In view of Remark 9 , I suggest to change it to "where $u \in U(n)$"

14. Section 4.1, The proof of Lemma 1, Sentence 1.

    "where u is such that $\sum_{i=1,\ldots,n} u_i = 0$". I recommend to change it to "where $u \in U(n)$".

15. Section 4.1, The proof of Lemma 1, penultimate sentence.

    divis or $\longrightarrow$ divisor

16. Section 4.1, The proof of Proposition 6, Sentence 1.

    is an $\theta_1(J(n, k))$-eigenvector $\longrightarrow$ is a $\theta_1(J(n, k))$-eigenvector

17. Section 4.1, The proof of Proposition 6, Sentence 1.

    $\sum_{i=1,\ldots,n} u_i = 0 \longrightarrow u \in U(n)$

18. Section 4.1, The proof of Proposition 7, Sentence 2.

    Let us describe $\longrightarrow$ Define

19. Section 4.1, The proof of Proposition 7, Paragraph 4, Sentence 2.

    Proposition $\longrightarrow$ proposition

20. Section 4.1, sentence before Lemma 2.

    Lemmas $\longrightarrow$ lemmas

21. Section 4.1, Statement of Lemma 2, Sentence 1.

    $(n, s) = 1 \longrightarrow (r, s) = 1$; and s is divisor of (n, k),. $\longrightarrow$ delete comma

    "Let u be a real-valued vector indexed by the vertices of the graph $J(n, 1)$ and $\sum_{i=1}^{n} u_i = 0$". In view of Remark 9, I recommend to change it to "Let u be a vector from $U(n)$"

22. Section 4.1, Statement of Lemma 2, Sentence 2.

    "If $W^T u$ is a NZI $\theta_1(J(n,k))$-eigenvector of $J(n,k)$". I suggest to change it to "If $W^T u$ is a NZI vector"

23. Section 4.1, Statement of Lemma 3, Sentence 1.

    "Let u be a real-valued vector indexed by the vertices of the graph $J(n,1)$, $\sum_{i=1}^{n} u_i = 0$". In view of Remark 9, I recommend to change it to "Let u be a vector from $U(n)$"

24. Section 4.1, The proof of Theorem 4, Sentence 1.

    the vector u, $\sum_{i=1,\dots,n} u_i = 0 \longrightarrow$ a vector $u \in U(n)$

25. Section 4.1, The proof of Corollary 1, Sentence 3.

    In the other hand $\longrightarrow$ On the other hand

26. Section 4.1, The proof of Corollary 2, Sentence 1.

    "Let u be the real-valued vector of the vertices of the graph $J(n,1)$ with the sum of values is zero,". In view of Remark 9, I recommend to change it to "Let u be a vector from $U(n)$"

27. Section 4.1, The proof of Corollary 2, Sentence 2.

    by Proposition 6 $\longrightarrow$ due to Proposition 6

28. Section 4.1, The proof of Proposition 8, Sentence 1.

    Let u be the real-valued vector $\longrightarrow$ Let u be a vector from $U(n)$

29. Section 4.1, The proof of Proposition 8, Sentence 3.

    In the other hand $\longrightarrow$ On the other hand

30. Section 4.1, The proof of Theorem 5, Sentence 1.

    Let u be the real-valued vector $\longrightarrow$ Let u be a vector from $U(n)$

31. Section 4.1, The proof of Theorem 5, Sentence 3.

    In what follows n is odd $\longrightarrow$ In what follows we assume that n is odd

32. Section 4.2, The proof of Theorem 6, Paragraph 1, Sentence 4.

    $\theta_1(J(n,3))$-eigenvector $\longrightarrow$ $\theta_1(\Gamma_S)$-eigenvector, by Theorem 3 $\longrightarrow$ due to Theorem 3

33. Section 4.2, The proof of Theorem 6, Paragraph 1, Sentence 6.

    $(W_S^T)_{i,j,k} \longrightarrow (W_S^T u)_{\{i,j,k\}}$

34. Section 4.2, The proof of Theorem 6, Paragraph 1, the last sentence.

    I see that $\|W_S^T u\|_\infty \leq 3$, but it is not clear that $\|W_S^T u\|_\infty = 3$. Please, give more details.

35. Section 4.2, The proof of Theorem 6, Paragraph 2, Sentence 4.

    I suggest to delete "STS".

36. Section 4.2, The proof of Theorem 6, Paragraph 5, Sentence 1.

    u is (-1)-eigenvector $\longrightarrow$ u is a (-1)-eigenvector

37. Section 4.2, The proof of Theorem 6, Paragraph 5, Sentence 2.

    $W^T u \longrightarrow W_S^T u$, $\theta_1(J(n,3)) \longrightarrow \theta_1(\Gamma_S)$, $J(n,3) \longrightarrow \Gamma_S$,

    by Theorem 3 $\longrightarrow$ due to Theorem 3

38. Section 4.2, The proof of Theorem 6, Paragraph 5, the last sentence.

    I see that $\|W_S^T u\|_\infty \leq 4$, but it is not clear that $\|W_S^T u\|_\infty = 4$. Please, give more details.

39. Section 5, Sentence 1. eigevectors $\longrightarrow$ eigenvectors.

40. Section 5.1, The proof of Proposition 9, Paragraph 1, Sentence 2.

    we see that 3-flow exists $\longrightarrow$ we see that a 3-flow exists

41. Section 5.1, The proof of Proposition 9, Paragraph 2, Sentence 1.

    "When r is odd, then it is easy to see that 2^r is 2 modulo 6 by induction on r". I suggest to delete "by induction on r".

42. Section 5.1, The proof of Proposition 9, Paragraph 4, Sentence 1.

    Steiner triple systems constructed by Bose method of order 9p were recently shown to be resolvable $\longrightarrow$ Steiner triple systems of order 9p constructed by Bose method were recently shown to be resolvable

43. Section 5.2, Paragraph 1, the definition of $\overline{S}$.

    It seems that $P'(\{i,j,k\})$ and $P(\{i,j,k\})$ for the case $\tau(\{i,j,k\}) = 1$ in the definition of $\overline{S}$ are inconsistent.

44. Section 5.2, Paragraph 2. "Our goal is to construct a zero-sum 5-flow in $S' = S^{2n+1,\tau}$". What is S'?

45. Section 5.2, The proof of Theorem 7, Sentence 1.

    from STS S $\longrightarrow$ from a STS S