Review of manuscript
"Validation via modified chi-square goodness of fit test for Poisson Topp Leone-G family of distributions with applications to failure time"
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1. If $H(x)$ is a cdf then

$$F(x) = \frac{1 - \exp(-\lambda H^{\alpha}(x))}{1 - e^{-\lambda}} \tag{1}$$

is a cdf for any $\lambda \neq 0$, $\alpha > 0$.

Authors follow Merovci et al. (2020) and study a very specific

$$F(x) = \frac{1 - \exp(-\lambda G^{\alpha}(x)(2 - G(x))^{\alpha})}{1 - e^{-\lambda}}, \tag{2}$$

that corresponds to

$$H(x) = G(x)(2 - G(x)) = 1 - \overline{G}^2(x) \tag{3}$$

with the tail $\overline{G}(x) = 1 - G(x)$ of cdf $G(x)$.

The concrete cdfs under authors' consideration are exponential, Weibull and Burr-XII ones, the last with $G(x) = 1 - (1 + x^{\delta})^{-\beta}$. From (3), the corresponding cdfs $H(x)$ are exponential, Weibull and Burr-XII ones too, with doubled corresponding parameters. For example, $H(x) = 1 - (1 + x^{\delta})^{-2\beta}$ for Burr-XII distribution.

Therefore one does not need the complicated concept (2) for these 3 examples of distributions. It is enough to use more simple formula (1).

2. Formula (8) in Chakraborty et al. (2020) gives

$$F^{KwPG}(x; a, b, \lambda) = 1 - \left(1 - \left(\frac{1 - e^{-\lambda G(x)}}{1 - e^{-\lambda}}\right)^{a}\right)^{b}.$$

So authors could substitute $G^{\alpha}(x)$ instead of $G(x)$ and have

$$F(x; a, b, \lambda, \alpha) = 1 - \left(1 - \left(\frac{1 - e^{-\lambda G^{\alpha}(x)}}{1 - e^{-\lambda}}\right)^{a}\right)^{b}. \tag{4}$$

This is the direct generalization of Chakraborty et al. (2020) and of formula (1) which corresponds to $a = b = 1$.

3. The content of Section 2 can be compactly rewritten for more general model (4). The calculations given in Section 2 are elementary. They contain a large number of typos and incorrectly positioned indexes. See, for example, lines 2, 8, 9 (2 different typos), 11, 13, 14 from the bottom of page 5. It would be better to use latex for typing formulas.

4. Calculations in Section 3 concern likelihood functions and Fisher information matrices for these 3 examples. There are elementary calculations

of derivatives with many typos and incorrectly positioned indexes. Section 3 contains results of simulations and parameter estimations but there is no code and no link to the program being used.

5. Authors apply the Nikulin-Rao-Robson test for these 3 examples in Section 4. The statistics of the test is given with a misprint (the last line on p. 10). There is a strange effect of simulations in Tables 3, 4, 5. Deviations of empirical levels from theoretical ones increase on n, sample sizes, for $n \geq 50$. The deviations are up to 90% for $n = 300$. Maybe the reason is a programming error or an incorrect choice of the number of intervals. Authors should describe the algorithm of choice of the number of intervals. Last lines in Tables 3 and 5 contain obvious typos.

6. Real data examples have very small sample sizes (84, 70, 72 in Section 4, 20 in Section 5). So they fit pretty well with any reasonable hypothesis. An example with $n \geq 200$ is needed.

7. Table 7 does not contain log-likelihood values. P-values for KS-test are not the real p-values because this test is not parametric.

I can recommend a major revision.