REPORT ON THE PAPER "SEQUENTIAL LABYRINTH FRACTALS".

1. Reference

This is a report on the paper "Sequential Labyrinth Fractals" by Harsha Gopalakrishnan and Srijanani Anurag Prasad, submitted to the journal Siberian Electronic Mathematical Reports.

2. Description of the paper

Nowadays, the word "fractal" sometimes is used in the mathematical literature to define a "set of fractional Hausdorff dimension". But how can one construct a fractal (say, with some specific properties) via an explicit construction ? For instance, the Cantor set which is (de facto) the first fractal ever considered is constructed explicitly. Several classical methods lead to this goal. Among them is the finite subdivision method, introduced by Bedford and McMullen (independently) in 1984.

Based on the Bedford and McMullen method, Cristea and Stiensky proposed the notion of labyrinth fractals. The name labyrinth fractals comes from the fact that those fractals resemble labyrinths. Notice that labyrinth fractals were constructed by the latter group of two authors explicitly as well. Indeed, let us describe in more detail the iterative process by which the classical labyrinth fractals are constructed. Let m be a natural number and let I denote the unit square. At each step of the process: first, divide the set from the previous iteration (at the first step it's I) into m^2 equal subsquares that form a grid. Then, choose a subfamily of these smaller subsquares (call them white) with certain properties, to be disclosed in a moment. Consider the graph whose vertices are the centers of the chosen white subsquares and where there is an edge between two vertices if and only if the corresponding subsquares have a common side. The following three properties are required from the chosen subfamily. The tree propery, i.e. that its graph is a tree. The exits property, i.e. that there is exactly one horizontal and exactly one vertical pair of sides among all white subsquares that are subsets of the boundary of the set from the previous iteration step. The corner property, i.e. that if there is a white square at a corner, then there is no white square at the diagonally opposite corner. Next, continue iteratively, in each of the subsquares with the same subfamilly choosen at each iteration.

Since the papers of Cristea and Stiensky were published, many different types of labyrinth fractals have been given in the literature. In the present paper, the authors give a *new interesting* type of labyrinth fractals, namely the one based on two sequences of reals. These reals play the role of division ratios at corresponding steps of the iteration. In more detail, at each step instead of dividing the set from the previous iteration into m^2 equal squares, the authors here divide it into a general union of rectangles with certain natural properties. The rest of the algorithm follows more or less closely the steps described in the previous paragraph.

Sets resulting from this construction are called in this paper *sequential labyrinth fractals.*

The sequential labyrinth fractals are the main new objects of the current paper ; their construction and properties are its main new results. For instance (among other things), the authors explain the difference between the fractals they construct and the classical labyrinth fractals, mixed labyrinth fractals and super-mixed labyrinth fractals. Moreover, in Theorems 4.1, 4.2 and 4.3 the authors estimate and calculate the Hausdorff and box-counting dimensions of the sequential labyrinth fractals they construct.

3. Remarks on the paper

I have some general comments on the paper. Firstly, it would be interesting to find out whether the main results and the approach of the present paper hold in the case of several dimensions. Secondly, bearing in mind the fact that the sequential labyrinth fractals are constructed via an explicit procedure, it would be interesting to calculate their Fourier and Assouad dimensions as well. But apparently, these two comments might be topics for future works.

Thirdly, I would like to point out that the paper is written a bit sloppily. There are two particular things in this direction that I would like to stress. First, English could be improved at many places. Second, the proofs must be provided with parallel text that clearly explains what is happening in them.

Finally, there are many typos in the text. I list them as well as some suggestions to the authors below.

(1) Page 1. Please, replace the first phrase in the chapter Introduction by "In the literature, there are various methods for construction fractals."

(2) Page 2, line 9 from above. Please, read "Those can be considered as generalizations of the Bedford–McMullen carpet."

(3) Page 3, the phrase right before Definition 2.1. Please, read "We let I denote the unit square $[0, 1] \times [0, 1]$ throughout this paper."

(4) Page 3. Please, write before Definition 2.1 at least one introductory phrase explaining the main idea/main meaning of this definition.

(5) Page 3. Please write before Definition 2.2 at least one introductory phrase explaining the main idea/main meaning of this definition.

(6) Page 3. Please write before Theorem 2.1 at least one introductory phrase explaining the main idea/main meaning of this theorem.

(7) Page 4, the first line from above. Please add the phrase "We have" after the word Proof.

(8) Page 4, the fourth line from below. Please, give a formal definition of the remainder function Rem.

(9) Page 4. Please write before Proposition 2.1 at least one introductory phrase explaining the main idea/main meaning of this proposition.

(10) Page 4, the last line. Please, read "The proof of Proposition 2.1 is easy. It is given in Appendix 5."

(11) Page 5. Please write before Definition 2.3 at least one introductory phrase explaining the main idea/main meaning of this definition.

(12) Page 5. Please write before Theorem 2.2 at least one introductory phrase explaining the main idea/main meaning of this theorem.

(13) Page 8. Please write before Theorem 2.4 at least one introductory phrase explaining the main idea/main meaning of this theorem.
(14) Page 12. Please write before Theorem 4.2 at least one introductory phrase explaining the main idea/main meaning of this theorem.
(15) Page 13. Please write before Theorem 4.3 at least one introductory phrase explaining the main idea/main meaning of this theorem.

4. Conclusion

To conclude, this is a good paper. The results are new and interesting to the scientific community. The paper is a research which belongs to Fractal Geometry. Being recent, it has already been cited several times in the literature. There are however many things in the paper that should be modified, and after that those things will have been taken into account, **I will recommend the publication** of this paper in Siberian Electronic Mathematical Reports.