

Referee report for the paper
“A Study of Axiomatizability And Decidability of the Mathematical Structures”
by Zahra Sheikholeslami
submitted to “Mathematical notes of NEFU”

Actually, the paper is a compilation of known results, easy-to-prove results, and incorrect statements. The paper should be rejected.

Here are only some remarks.

- On p. 1, the author claims to answer the question (marked as Main Problem 1), whether the structure $\langle Q; \sqsubseteq \rangle$, where \sqsubseteq is the divisibility relation, is decidable. Taking into account that $x \sqsubseteq y$ (x divides y , i.e. $\exists t(xt = y)$) in Q (rational numbers) is equivalent to $x = 0 \rightarrow y = 0$, all the formulas of the language \sqsubseteq can be effectively transformed into equivalent formulas of the language of the signature consisting of a single constant 0. Quantifier elimination for this language is known (and is an easy exercise). It follows that the theory of $\langle Q; \sqsubseteq \rangle$ is decidable.
- Further on, the author claims to prove quantifier elimination for atomic Boolean algebras, which is a false statement, because one can easily ascertain that the theory of Boolean algebras, T_{BA} , is not submodel complete in the standard signature: let D be the diagram of a subalgebra generated by some $a \neq 0, 1$; then $T_{BA} + D$ is not complete: it does not answer neither the first order question “is a an atom?” nor its negation. Although, atomic Boolean algebras are known to admit quantifier elimination in some extended signatures.
- The paper also contains too much full proofs of well-known statements accompanied with references to appropriate literature without special reasons, which is not ‘comme il faut’ for research papers. So are, for example, Theorems 4,5,13 and more.
- For Theorem 9, there is no neither proof nor reference to a proof.
- Theorem 10 is a well-known result that can be found in many books on model theory for beginners.
- Theorem 11. The result easily follows from submodel completeness of this theory.
- Theorem 14 says that the theory of $\langle R; < \rangle$ admits quantifier elimination. One does not need to give an extra proof just because this is a well-known theory of dense linear orderings without endpoints, for which the result is known.

Actually, I can give much more remarks on the paper. I understand that the author will be unhappy of my reference; but I wish her not to give up.

Referee