Referee report

Title: On nilpotent Schur groups

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In this paper the author considers the Schur groups, a class of finite groups defined by Pöschel [10] in the context of Schur rings. Pöschel posed the question as what kind of structure a Schur group can have. This question has attracted considerable attention and it is still open. The introduction contains a very good update on the current status of the problem.

The main result is a reduction of the class of nilpotent Schur groups into a restrictive list. In my opinion, this is an important step towards the complete answer to Pöschel's question. The result is new and non-trivial. It is based on some beautiful constructions and a clever argument to show that a nilpotent Schur group has a restrictive structure.

The paper is clearly written. I went through the computations and found them to be correct up to some minor inaccuracies, see the comments below. In my opinion, this is a solid contribution to the theory of Schur groups, therefore, I suggest its publication.

Specific comments:

1. Page 2, line 15: "one of the explicitly given families". Maybe it's better to write "one of a few explicitly given families".

2. Page 3, Lemma 2.1: $X^{(m)}$ in the lemma is not defined. It's better to define it somewhere before the lemma.

3. Page 4, line 5: "then $\alpha^{G/A}$ is denoted the" to "then $\alpha^{G/A}$ denotes the".

4. Page 4, line 13: "Then $\mathrm{Orb}(K, G)$ forms a partition of G that defines the S-ring ..." to "Then $\mathrm{Orb}(K, G)$ forms a partition of G that defines an S-ring ...".

5. Page 4, proof of Lemma 3.2: I agree that $\mathcal{A}$ is an S-ring. But I cannot understand what is $\mathrm{Span}\{\underline{T_0} : T_0 \subseteq H, c^{i+j}T_0 \in \mathcal{S}\}$. I checked it myself by observing that merging X_3 with X_4 and Y_3 with Y_4 yields an S-ring. Similar idea as in the proof of Lemma 4.4.

   Please, make sure that in the product $\underline{XY}$, there is a space between the factors. It's better to write like this $\underline{X}\,\underline{Y}$. Please, also check other places.

6. Page 5, line 8 in the proof of Lemma 3.4: For the intersection $X_3 a c^3 \cap A_1 a c^3$, I think that $X_3 a c^3$ should be replaced with "$X_3 c^2$".

7. Page 5, line -8: "is a generator of $\mathrm{Aut}(C)$ acting ..." to "is the permutation induced by a generator of $\mathrm{Aut}(C)$ acting ...".

8. Page 5, formula (3): I think that $c^{k+1}_{i+1,j+1}$ can also be written for $c^{kf}_{if,jf}$ with the remark that additions are done modulo 6. This might be easier to use.

9. Page 9, line 14: "... and Eqs. (15) and (16), we obtain $t = 2u$." Please, recheck this. I got a different result.

10. Page 9, line -6: "$C_1 = \langle c_1 \rangle \cong C_p$" to "$C_1 = \langle c_1 \rangle \cong \mathbb{Z}_p$".

11. Page 9, line -3: "$\mathcal{A}$ in the $(A \times C)/A_1$-wreath product" to "$\mathcal{A}$ is the $(A \times C)/A_1$-wreath product".