NONLINEAR *-JORDAN-TYPE DERIVATIONS ON ALTERNATIVE *-ALGEBRAS

A.J.O. ANDRADE, G.C. MORAES, R.N. FERREIRA, B.L.M. FERREIRA

ABSTRACT. Let A be an unital alternative *-algebra. Assume that A contains a nontrivial symmetric idempotent element e which satisfies $xA \cdot e = 0$ implies $x = 0$ and $xA \cdot (1_A - e) = 0$ implies $x = 0$. In this paper, it is shown that Φ is a nonlinear *-Jordan-type derivation on A if and only if Φ is an additive *-derivation. As application, we get a result on alternative W^*-algebras.

Keywords: *-Jordan-type derivation, *-derivation, alternative *-algebras.

INTRODUCTION

An algebra A not necessarily associative or commutative is called an alternative algebra if it satisfies the identities $a^2 b = a(ab)$ and $ba^2 = (ba)a$, for all elements $a, b \in A$. One easily sees that any associative algebra is an alternative algebra. An alternative algebra A is called prime if for any elements $a, b \in A$ satisfying the condition $aAb = 0$, then either $a = 0$ or $b = 0$. Let A and B be alternative algebras. We say that a mapping $\Phi : A \to B$ preserves product if $\Phi(ab) = \Phi(a)\Phi(b)$, for all elements $a, b \in A$, and that it preserves Jordan product (resp., preserves Lie product) if $\Phi(ab + ba) = \Phi(a)\Phi(b) + \Phi(b)\Phi(a)$ (resp., $\Phi(ab - ba) = \Phi(a)\Phi(b) - \Phi(b)\Phi(a)$), for all elements $a, b \in A$. We say that a mapping $\Phi : A \to B$ is additive if $\Phi(a+b) = \Phi(a)+\Phi(b)$, for all elements $a, b \in A$ and that it is an isomorphism if Φ is an additive bijection that preserves products. By involution, we mean a mapping $* : A \to A$ such that $(x+y)^* = x^* + y^*$; $(x^*)^* = x$ and $(xy)^* = y^* x^*$ for all $x, y \in A$. An element $s \in A$ satisfying $s^* = s$ is called a symmetric element of A. Let A and B

be alternative $*$-algebras. We say that a mapping $\Phi : A \to B$ preserves involution if $\Phi(a^*) = \Phi(a)^*$, for all elements $a \in A$, and that Φ is a $*$-isomorphism if Φ is an isomorphism that preserves involution.

Let A be an alternative $*$-algebra over the ground field $\mathbb{C}$. For any $a, b \in A$ denote a new product of a and b by $a \bullet b = ab + ba^*$, and this new product $\bullet$ is usually known as Jordan $\bullet$-product. Such kind of product plays a more and more important role in some research topics, and its study has attracted many authors' attention. Particular attention has been paid to understanding maps that preserve the product $AB + BA^*$ on $*$-algebra (for example, see [3, 14, 16, 17, 18, 25]). The product is extensively studied because it naturally arises in the problem of representing quadratic functionals with sesquilinear functionals (for example, see [20, 21, 21]) and in the problem of characterizing ideals (for example, see [1, 19]). Furthermore, there has been a great interest in the study of additivity or characterization of mappings that preserve some kind of product on non-associative rings and algebras for example in the paper's [7, 10, 8, 5, 2]. For these kinds of mappings defined above, in a recent paper, Ferreira and Costa [6] has studied the characterization about $*$-Jordan-type maps on C^*-algebras, they have proved under mild conditions that every multiplicative $*$-Jordan-type maps are $*$-isomorphism. That work motivated us to study the same question for the case $*$-Jordan-type derivations on a class of nonassociative algebras, namely $*$-alternative algebras. Let us define $*$-Jordan-type derivations.

Recall that an additive map $\Phi : A \to A$ is said to be an additive derivation if $\Phi(ab) = \Phi(a)b + a\Phi(b)$ for all $a, b \in A$ Furthermore, Φ is said to be an additive $*$-derivation if it is an additive derivation and satisfies $\Phi(a^*) = \Phi(a)^*$ for all $a \in A$. We say that $\Xi_{y,z} : A \to A$ is an additive $*$-inner derivation if it is an additive inner derivation, that is,

$$\Xi_{y,z}(a) = ([L_y, L_z] + [L_y, R_z] + [R_y, R_z])(a),$$

is holds true for all $y, z \in A$ and satisfies $\Xi(a^*) = \Xi(a)^*$. A not necessarily linear map $\Phi : A \to A$ is said to be a nonlinear $*$-Jordan derivation if

$$\Phi(a \bullet b) = \Phi(a) \bullet b + a \bullet \Phi(b)$$

for all $a, b \in A$. Similarly, a map $\Phi : A \to A$ is said to be a nonlinear $*$-Jordan triple derivation if

$$\Phi(a \bullet b \bullet c) = \Phi(a) \bullet b \bullet c + a \bullet \Phi(b) \bullet c + a \bullet b \bullet \Phi(c)$$

for all $a, b, c \in A$, where $a \bullet b \bullet c = (a \bullet b) \bullet c$ (we should be aware that $\bullet$ is not necessarily associative). Given the consideration of nonlinear $*$-Jordan derivations and nonlinear $*$- Jordan triple derivations, as well as introduced the definition of $*$-Jordan-type maps in [6] we introduce the following. Suppose that $n \geq 2$ is a fixed positive integer. Accordingly, a nonlinear $*$-Jordan n-derivation is a map $\Phi : A \to A$ satisfying the condition

$$(1) \qquad \Phi(a_1 \bullet a_2 \bullet \cdots \bullet a_n) = \sum_{k=1}^{n} a_1 \bullet \cdots \bullet a_{k-1} \bullet \Phi(a_k) \bullet a_{k+1} \bullet \cdots \bullet a_n$$

for all $a_1, a_2, \ldots a_n \in A$, where $a_1 \bullet a_2 \bullet \cdots \bullet a_n = (\cdots((a_1 \bullet a_2) \bullet a_3) \cdots \bullet a_n)$. By the definition, it is clear that every $*$-Jordan derivation is a $*$-Jordan 2-derivation and every $*$-Jordan triple derivation is a $*$-Jordan 3-derivation. It is obvious that every nonlinear $*$-Jordan derivation on any $*$-algebra is a $*$-Jordan n-derivation.

But we do not know whether the converse is true. ∗-Jordan 2-derivation, ∗-Jordan 3-derivation, and ∗-Jordan n-derivation are collectively referred to as ∗-Jordan-type derivations. ∗-Jordan-type derivations in different backgrounds are extensively studied by several authors, see [4, 15, 22, 23, 24].

It is well known that A has a Peirce decomposition $A = A_{11} \oplus A_{12} \oplus A_{21} \oplus A_{22}$, where $A_{ij} = e_i A e_j$ $(i, j = 1, 2)$ [13], satisfying the following multiplicative relations:

(i) $A_{ij} A_{jl} \subseteq A_{il}$ $(i, j, l = 1, 2)$;
(ii) $A_{ij} A_{ij} \subseteq A_{ji}$ $(i, j = 1, 2)$;
(iii) $A_{ij} A_{kl} = 0$, if $j \neq k$ and $(i, j) \neq (k, l)$, $(i, j, k, l = 1, 2)$;
(iv) $x_{ij}^2 = 0$, for all $x_{ij} \in A_{ij}$ $(i, j = 1, 2; \ i \neq j)$.

The notion of Peirce decomposition for alternative algebras is similar to that one for associative algebras. However, this similarity is restricted to its written form, not including its theoretical structure since Peirce decomposition for alternative algebras is a generalization of that classical one for associative algebras.

1. Main theorem

We shall prove as follows the main result of this paper.

Theorem 1. *Let A be a unital alternative ∗-algebra with the unit 1_A. Assume that A contains a nontrivial symmetric idempotent element e which satisfies*

(♠) $xA \cdot e = 0$ *implies* $x = 0$;
(♣) $xA \cdot (1_A - e) = 0$ *implies* $x = 0$.

If a map $\Phi : A \to A$ satisfies

$$\Phi(a_1 \bullet a_2 \bullet \cdots \bullet a_n) = \sum_{k=1}^{n} a_1 \bullet \cdots \bullet a_{k-1} \bullet \Phi(a_k) \bullet a_{k+1} \bullet \cdots \bullet a_n$$

for all $a_{n-1}, a_n \in A$ and $a_i = 1_A$ for all $i \in \{1, 2, \ldots, n-2\}$, then Φ is an additive ∗-derivation.

Corollary 2. *Let A be a unital alternative ∗-algebra with the unit 1_A. Assume that A contains a nontrivial symmetric idempotent element e which satisfies (♠) and (♣). Then Φ is a nonlinear ∗-Jordan-type derivation on A if and only if Φ is an additive ∗-derivation.*

It is obvious that prime alternative ∗-algebras satisfy the assumptions "$xA \cdot e = 0 \Rightarrow x = 0$ and $xA \cdot (1_A - e) = 0 \Rightarrow x = 0$". Thus, we have the following result

Corollary 3. *Let A be a prime unital alternative ∗-algebra with the unit 1_A. Assume that A contains a nontrivial symmetric idempotent element e. If a map $\Phi : A \to A$ satisfies*

$$\Phi(a_1 \bullet a_2 \bullet \cdots \bullet a_n) = \sum_{k=1}^{n} a_1 \bullet \cdots \bullet a_{k-1} \bullet \Phi(a_k) \bullet a_{k+1} \bullet \cdots \bullet a_n$$

for all $a_{n-1}, a_n \in A$ and $a_i = 1_A$ for all $i \in \{1, 2, \ldots, n-2\}$, then Φ is an additive ∗-derivation.

Remark 4. *Some examples of non-prime alternative algebras satisfying the assumptions "$xA \cdot e = 0 \Rightarrow x = 0$ and $xA \cdot (1_A - e) = 0 \Rightarrow x = 0$"were given in [9].*

2. The proof of main result

Consider the Peirce decomposition of A to respect nontrivial symmetric idempotent element e. In all that follows, when we write a_{ij}, it indicates that $a_{ij} \in A_{ij}$ and $e_1 = e$, $e_2 = 1_A - e_1$. We will complete the proof by proving several claims.

Borrowing the notation in [8] let us point out a non-associative monomial m of degree n by the following expression form

$$m(x_1, x_2, \cdots, x_n, x_{n-1}) := ((\cdots (x_1 \bullet x_2) \cdots) \bullet x_n).$$

When the first $n - 2$ variables in the non-associative monomial m assume equal values, we will denote by

$$m(\underbrace{a, \cdots, a}_{n-2}, x_{n-1}, x_n) := \xi_a(x_{n-1}, x_n), \quad \text{for any } a \in A.$$

Claim 5. $\Phi(0) = 0$.

Indeed,

$$\Phi(0) = \Phi(\xi_{1_A}(0, 0)) = 0.$$

Claim 6. *For every* $a_{12} \in A_{12}$, $b_{21} \in A_{21}$ *we have*

$$\Phi(a_{12} + b_{21}) = \Phi(a_{12}) + \Phi(b_{21}).$$

Consider $t = \Phi(a_{12} + b_{21}) - \Phi(a_{12}) - \Phi(b_{21})$. Since

$$\xi_{1_A}((e_1 - e_2), a_{12}) = \xi_{1_A}((e_1 - e_2), b_{21}) = 0.$$

It follows from Claim 5 that

$$\sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(e_1 - e_2, a_{12} + b_{21}) + \xi_{1_A}(\Phi(e_1 - e_2), (a_{12} + b_{21}))$$

$$+ \xi_{1_A}((e_1 - e_2), \Phi(a_{12} + b_{21}))$$

$$= \Phi(\xi_{1_A}(e_1 - e_2, a_{12} + b_{21}))$$

$$= \Phi(\xi_{1_A}(e_1 - e_2, a_{12})) + \Phi(\xi_{1_A}(e_1 - e_2, b_{21}))$$

$$= \sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(e_1 - e_2, a_{12} + b_{21}) + \xi_{1_A}(\Phi(e_1 - e_2), (a_{12} + b_{21}))$$

$$+ \xi_{1_A}(e_1 - e_2, \Phi(a_{12}) + \Phi(b_{21})).$$

From this, we get that $\xi_{1_A}(e_1 - e_2, t) = 0$, so $t_{11} = t_{22} = 0$. Since $\xi_{1_A}(a_{12}, e_1) = 0$, we have

$$\xi_{1_A}(a_{12} + b_{21}, e_1) = \xi_{1_A}(b_{21}, e_1).$$

It follows that

$$\sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(a_{12} + b_{21}, e_1) + \xi_{1_A}(\Phi(a_{12} + b_{21}, e_1)) + \xi_{1_A}(a_{12} + b_{21}, \Phi(e_1))$$

$$= \Phi(\xi_{1_A}(a_{12} + b_{21}, e_1))$$

$$= \Phi(\xi_{1_A}(a_{12}, e_1)) + \Phi(\xi_{1_A}(b_{21}, e_1))$$

$$= \sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(a_{12} + b_{21}, e_1) + \xi_{1_A}(\Phi(a_{12}) + \Phi(b_{21}), e_1) + \xi_{1_A}(a_{12} + b_{21}, e_1).$$

Hence $\xi_{1_A}(t, e_1) = 0$ from which we get that $t_{21} = 0$. Similarly, $t_{12} = 0$, proving the claim.

Claim 7. *For every $a_{11} \in A_{11}$, $b_{12} \in A_{12}$, $c_{21} \in A_{21}$, $d_{22} \in A_{22}$ we have*

$$\Phi(a_{11} + b_{12} + c_{21}) = \Phi(a_{11}) + \Phi(b_{12}) + \Phi(c_{21})$$

and

$$\Phi(b_{12} + c_{21} + d_{22}) = \Phi(b_{12}) + \Phi(c_{21}) + \Phi(d_{22}).$$

Let $t = \Phi(a_{11} + b_{12} + c_{21}) - \Phi(a_{11}) - \Phi(b_{12}) - \Phi(c_{21})$. Since $\xi_{1_A}(e_2, a_{11}) = 0$ it follows from Claim 6 that

$$
\begin{aligned}
\sum_{\Phi(1_A)} & \xi_{\Phi(1_A)}(e_2, a_{11} + b_{12} + c_{21}) + \xi_{1_A}(\Phi(e_2), a_{11} + b_{12} + c_{21}) \\
& + \xi_{1_A}(e_2, \Phi(a_{11} + b_{12} + c_{21})) \\
& = \Phi(\xi_{1_A}(e_2, a_{11} + b_{12} + c_{21})) \\
& = \Phi(\xi_{1_A}(e_2, a_{11})) + \Phi(\xi_{1_A}(e_2, b_{12} + c_{21})) \\
& = \Phi(\xi_{1_A}(e_2, a_{11})) + \Phi(\xi_{1_A}(e_2, b_{12})) + \Phi(\xi_{1_A}(e_2, c_{21})) \\
& = \sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(e_2, a_{11} + b_{12} + c_{21}) + \xi_{1_A}(\Phi(e_2), a_{11} + b_{12} + c_{21}) \\
& + \xi_{1_A}(e_2, \Phi(a_{11}) + \Phi(b_{12}) + \Phi(c_{21})).
\end{aligned}
$$

Hence $\xi_{1_A}(e_2, t) = 0$ from which we get that $t_{12} = t_{21} = t_{22} = 0$. Since

$$\xi_{1_A}(e_1 - e_2, a_{12}) = \xi_{1_A}(e_1 - e_2, a_{21}) = 0$$

one has

$$
\begin{aligned}
\sum_{\Phi(1_A)} & \xi_{\Phi(1_A)}(e_1 - e_2, a_{11} + b_{12} + c_{21}) + \xi_{1_A}(\Phi(e_1 - e_2), a_{11} + b_{12} + c_{21}) \\
& + \xi_{1_A}(e_1 - e_2, \Phi(a_{11} + b_{12} + c_{21})) \\
& = \Phi(\xi_{1_A}(e_1 - e_2, a_{11} + b_{12} + c_{21})) \\
& = \Phi(\xi_{1_A}(e_1 - e_2, a_{11})) + \Phi(\xi_{1_A}(e_1 - e_2, b_{12})) + \Phi(\xi_{1_A}(e_1 - e_2, c_{21})) \\
& = \sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(e_1 - e_2, a_{11} + b_{12} + c_{21}) + \xi_{1_A}(\Phi(e_1 - e_2), a_{11} + b_{12} + c_{21}) \\
& + \xi_{1_A}(e_1 - e_2, \Phi(a_{11}) + \Phi(b_{12}) + \Phi(c_{21}))
\end{aligned}
$$

from which we get that $\xi_{1_A}(e_1 - e_2, t) = 0$. Thus, $t_{11} = 0$, and then $t = 0$. Similarly, we can prove that $\Phi(b_{12} + c_{21} + d_{22}) = \Phi(b_{12}) + \Phi(c_{21}) + \Phi(d_{22})$.

Claim 8. *For every $a_{11} \in A_{11}$, $b_{12} \in A_{12}$, $c_{21} \in A_{21}$, $d_{22} \in A_{22}$ we have*

$$\Phi(a_{11} + b_{12} + c_{21} + d_{22}) = \Phi(a_{11}) + \Phi(b_{12}) + \Phi(c_{21}) + \Phi(d_{22}).$$

Let $t = \Phi(a_{11} + b_{12} + c_{21} + d_{22}) - \Phi(a_{11}) - \Phi(b_{12}) - \Phi(c_{21}) - \Phi(d_{22})$. Since $\xi_{1_A}(e_2, a_{11}) = 0$ it follows from Claim 7 that

$$\sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(e_2, a_{11} + b_{12} + c_{21} + d_{22}) + \xi_{1_A}(\Phi(e_2, a_{11} + b_{12} + c_{21} + d_{22}))$$

$$+ \xi_{1_A}(e_2, \Phi(a_{11} + b_{12} + c_{21} + d_{22}))$$

$$= \Phi(\xi_{1_A}(e_2, a_{11} + b_{12} + c_{21} + d_{22}))$$

$$= \Phi(\xi_{1_A}(e_2, a_{11})) + \Phi(\xi_{1_A}(e_2, b_{12} + c_{21} + d_{22}))$$

$$= \Phi(\xi_{1_A}(e_2, a_{11})) + \Phi(\xi_{1_A}(e_2, b_{12})) + \Phi(\xi_{1_A}(e_2, c_{21})) + \Phi(\xi_{1_A}(e_2, d_{22}))$$

$$= \sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(e_2, a_{11} + b_{12} + c_{21} + d_{22}) + \xi_{1_A}(\Phi(e_2), a_{11} + b_{12} + c_{21} + d_{22})$$

$$+ \xi_{1_A}(e_2, \Phi(a_{11}) + \Phi(b_{12}) + \Phi(c_{21}) + \Phi(d_{22})).$$

Hence $\xi_{1_A}(e_2, t) = 0$ from which we get that $t_{12} = t_{21} = t_{22} = 0$. Similarly, we can prove $t_{11} = 0$ proving the claim.

In the next Claim, we have a slight difference from the case of associative algebras. In the associative case we know that $a_{ij}b_{ij} = 0$ for $i \neq j$ but in general in alternative algebras we do not have this property.

Claim 9. *For every $a_{12}, b_{12} \in A_{12}$ and $c_{21}, d_{21} \in A_{21}$ we have*

$$\Phi(a_{12}b_{12} + a_{12}^*) = \Phi(a_{12}b_{12}) + \Phi(a_{12}^*)$$

and

$$\Phi(c_{21}d_{21} + c_{21}^*) = \Phi(c_{21}d_{21}) + \Phi(c_{21}^*).$$

Since

$$\xi_{1_A}\left(a_{12}, \frac{e_2 + b_{12}}{2^n}\right) = a_{12} + a_{12}b_{12} + a_{12}^* + b_{12}a_{12}^*$$

we get from Claim 8 that

$$\Phi(a_{12}) + \Phi(a_{12}b_{12} + a_{12}^*) + \Phi(b_{12}a_{12}^*)$$

$$= \Phi(a_{12} + a_{12}b_{12} + a_{12}^* + b_{12}a_{12}^*)$$

$$= \Phi\left(\xi_{1_A}\left(a_{12}, \frac{e_2 + b_{12}}{2^n}\right)\right)$$

$$= \sum_{\Phi(1_A)} \xi_{\Phi(1_A)}\left(a_{12}, \frac{e_2 + b_{12}}{2^n}\right) + \xi_{1_A}\left(\Phi(a_{12}), \frac{e_2 + b_{12}}{2^n}\right)$$

$$+ \xi_{1_A}\left(a_{12}, \Phi\left(\frac{e_2 + b_{12}}{2^n}\right)\right)$$

$$= \sum_{\Phi(1_A)} \xi_{\Phi(1_A)}\left(a_{12}, \frac{e_2 + b_{12}}{2^n}\right) + \xi_{1_A}\left(\Phi(a_{12}), \frac{e_2 + b_{12}}{2^n}\right)$$

$$+ \xi_{1_A}\left(a_{12}, \Phi\left(\frac{e_2}{2^n}\right) + \Phi\left(\frac{b_{12}}{2^n}\right)\right)$$

$$= \Phi\left(\xi_{1_A}\left(a_{12}, \frac{e_2}{2^n}\right)\right) + \Phi\left(\xi_{1_A}\left(a_{12}, \frac{b_{12}}{2^n}\right)\right)$$

$$= \Phi(a_{12}) + \Phi(a_{12}^*) + \Phi(a_{12}b_{12}) + \Phi(b_{12}a_{12}^*),$$

which implies $\Phi(a_{12}b_{12} + a_{12}^*) = \Phi(a_{12}b_{12}) + \Phi(a_{12}^*)$. Similarly, we prove the other case using the identity

$$\xi_{1_A}\left(c_{21}, \frac{e_1 + d_{21}}{2^n}\right) = c_{21} + c_{21}d_{21} + c_{21}^* + d_{21}c_{21}^*.$$

Claim 10. *For all $a_{ij}, b_{ij} \in A_{ij}$, $1 \leq i \neq j \leq 2$, we have*

$$\Phi(a_{ij} + b_{ij}) = \Phi(a_{ij}) + \Phi(b_{ij}).$$

Since

$$\xi_{1_A}\left(\frac{e_i + a_{ij}}{2^n}, e_j + b_{ij}\right) = a_{ij} + b_{ij} + a_{ij}^* + a_{ij}b_{ij} + b_{ij}a_{ij}^*$$

we get from Claim 9 that

$$\Phi(a_{ij} + b_{ij}) + \Phi(a_{ij}^* + a_{ij}b_{ij}) + \Phi(b_{ij}a_{ij}^*)$$

$$= \Phi\left(\xi_{1_A}\left(\frac{e_i + a_{ij}}{2^n}, e_j + b_{ij}\right)\right)$$

$$= \sum_{\Phi(1_A)} \xi_{\Phi(1_A)}\left(\frac{e_i + a_{ij}}{2^n}, e_j + b_{ij}\right) + \xi_{1_A}\left(\Phi\left(\frac{e_i + a_{ij}}{2^n}\right), e_j + b_{ij}\right)$$

$$\quad + \xi_{1_A}\left(\frac{e_i + a_{ij}}{2^n}, \Phi(e_j + b_{ij})\right)$$

$$= \sum_{\Phi(1_A)} \xi_{\Phi(1_A)}\left(\frac{e_i}{2^n} + \frac{a_{ij}}{2^n}, e_j + b_{ij}\right) + \xi_{1_A}\left(\Phi\left(\frac{e_i}{2^n}\right) + \Phi\left(\frac{a_{ij}}{2^n}\right), e_i + b_{ij}\right)$$

$$\quad + \xi_{1_A}\left(\frac{e_i}{2^n} + \frac{a_{ij}}{2^n}, \Phi(e_j) + \Phi(b_{ij})\right)$$

$$= \Phi\left(\xi_{1_A}\left(\frac{e_i}{2^n}, e_j\right)\right) + \Phi\left(\xi_{1_A}\left(\frac{a_{ij}}{2^n}, b_{ij}\right)\right) + \Phi\left(\xi_{1_A}\left(\frac{a_{ij}}{2^n}, e_j\right)\right)$$

$$\quad + \Phi\left(\xi_{1_A}\left(\frac{e_i}{2^n}, b_{ij}\right)\right)$$

$$= \Phi(b_{ij}) + \Phi(a_{ij} + a_{ij}^*) + \Phi(a_{ij}b_{ij} + b_{ij}a_{ij}^*)$$

$$= \Phi(b_{ij}) + \Phi(a_{ij}) + \Phi(a_{ij}^*) + \Phi(a_{ij}b_{ij}) + \Phi(b_{ij}a_{ij}^*),$$

which implies $\Phi(a_{ij} + b_{ij}) = \Phi(b_{ij}) + \Phi(a_{ij})$.

Claim 11. *For every $a_{ii}, b_{ii} \in A_{ii}$, $1 \leq i \leq 2$ we have*

$$\Phi(a_{ii} + b_{ii}) = \Phi(a_{ii}) + \Phi(b_{ii}).$$

Let $t = \Phi(a_{ii} + b_{ii}) - \Phi(a_{ii}) - \Phi(b_{ii})$. Since

$$\xi_{1_A}(e_j, a_{ii}) = \xi_{1_A}(e_j, b_{ii}) = 0,$$

it follows that

$$\sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(e_j, a_{ii} + b_{ii}) + \xi_{1_A}(\Phi(e_j), a_{ii} + b_{ii}) + \xi_{1_A}(e_j, \Phi(a_{ii} + b_{ii}))$$

$$= \Phi(\xi_{1_A}(e_j, a_{ii} + b_{ii}))$$

$$= \Phi(\xi_{1_A}(e_j, a_{ii})) + \Phi(\xi_{1_A}(e_j, b_{ii}))$$

$$= \sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(e_j, a_{ii} + b_{ii}) + \xi_{1_A}(\Phi(e_j), a_{ii} + b_{ii}) + \xi_{1_A}(e_j, \Phi(a_{ii}) + \Phi(b_{ii})).$$

Hence $\xi_{1_A}(e_j, t) = 0$ which implies $t_{ij} = t_{ji} = t_{jj} = 0$. Now we get $t = t_{ii}$. For every $c_{ij} \in A_{ij}$, $i \neq j$, it follows from Claim 10 that

$$\sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(a_{ii} + b_{ii}, c_{ij}) + \xi_{1_A}(a_{ii} + b_{ii}, c_{ij}) + \xi_{1_A}(a_{ii} + b_{ii}, \Phi(c_{ij}))$$

$$= \Phi(\xi_{1_A}(a_{ii} + b_{ii}, c_{ij}))$$

$$= \Phi(\xi_{1_A}(a_{ii}, c_{ij})) + \Phi(\xi_{1_A}(b_{ii}, c_{ij}))$$

$$= \sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(a_{ii} + b_{ii}, c_{ij}) + \xi_{1_A}(\Phi(a_{ii}) + \Phi(b_{ii}), c_{ij}) + \xi_{1_A}(a_{ii} + b_{ii}, \Phi(c_{ij})).$$

Hence $\xi_{1_A}(t_{ii}, c_{ij}) = 0$. Then $t_{ii}c_{ij} = 0$ for all $c_{ij} \in A_{ij}$, that is, $t_{ii}c \cdot e_j = 0$ for all $c \in A$. It follows from (♠) and (♣) that $t = t_{ii} = 0$, proving the claim.

Claim 12. Φ *is additive.*

Let $a = \sum_{i,j=1}^2 a_{ij}$, $b = \sum_{i,j=1}^2 b_{ij} \in A$. By Claim 9, Claim 10 and Claim 11, we have

$$\Phi(a + b) = \Phi\left(\sum_{i,j=1}^2 a_{ij} + \sum_{i,j=1}^2 b_{ij}\right) = \Phi\left(\sum_{i,j=1}^2 (a_{ij} + b_{ij})\right)$$

$$= \sum_{i,j=1}^2 \Phi(a_{ij} + b_{ij})$$

$$= \sum_{i,j=1}^2 \Phi(a_{ij}) + \sum_{i,j=1}^2 \Phi(b_{ij})$$

$$= \Phi\left(\sum_{i,j=1}^2 a_{ij}\right) + \Phi\left(\sum_{i,j=1}^2 b_{ij}\right) = \Phi(a) + \Phi(b).$$

Claim 13. $\Phi(1_A) \bullet a = 0$ *for all* $a \in A$.

Since $\Phi(1_A) \bullet 1_A = \Phi(1_A) + \Phi(1_A)^*$ is self-adjoint element, we have

$$2^{n-2}\Phi(1_A) = \Phi(\xi_{1_A}(1_A, 1_A))$$

$$= \sum_{\Phi(1_A)} \xi_{\Phi(1_A)}(1_A, 1_A) + \xi_{1_A}(\Phi(1_A), 1_A) + \xi_{1_A}(1_A, \Phi(1_A))$$

$$= (n-1)2^{n-2}(\Phi(1_A) \bullet 1_A) + 2^{n-1}\Phi(1_A),$$

which implies $\Phi(1_A) \bullet 1_A = 0$. For every $a \in A$, it follows that

$$2^{n-1}\Phi(a) = \Phi(\xi_{1_A}(1_A, a))$$

$$= \xi_{1_A}(\Phi(1_A), a) + \xi_{1_A}(1_A, \Phi(a))$$

$$= 2^{n-2}\Phi(1_A) \bullet a + 2^{n-1}\Phi(a).$$

Thus $\Phi(1_A) \bullet a = 0$.

Claim 14. $\Phi(e_i) = \Phi(e_i)^*$, $i = 1, 2$.

For every $a = a^* \in A$, since

$$\xi_{1_A}(a, e_i) = \xi_{1_A}(e_i, a),$$

we have
$$\Phi(\xi_{1_A}(a, e_i)) = \Phi(\xi_{1_A}(e_i, a)).$$
It follows from Claim 13 that
$$\xi_{1_A}(\Phi(a), e_i) + \xi_{1_A}(a, \Phi(e_i)) = \xi_{1_A}(\Phi(e_i, a)) + \xi_{1_A}(e_i, \Phi(a))$$
Then it is easy to compute that
$$a(\Phi(e_i) - \Phi(e_i)^*) = e_i(\Phi(a) - \Phi(a)^*).$$
We multiply above equation by e_j from left side, it follows that
$$e_j \cdot a(\Phi(e_i) - \Phi(e_i)^*) = 0$$
for all $a = a^* \in A$. Thus for every $b \in A$, since $b = b_1 + ib_2$ with $b_1 = \frac{b+b^*}{2}$ and $b_2 = \frac{b-b^*}{2i}$, we have $e_j \cdot b(\Phi(e_i) - \Phi(e_i)^*) = 0$ for all $b \in A$. Thus
$$(\Phi(e_i)^* - \Phi(e_i))b \cdot e_j = 0$$
for all $b \in A$. It follows from (♠) and (♣) that $\Phi(e_i) = \Phi(e_i)^*$.

Claim 15. $\Phi(e_i) = e_1 \Phi(e_i) e_2 + e_2 \Phi(e_i) e_1$ $(i = 1, 2)$ and $\Phi(1_A) = 0$.

On the one hand, by Claim 13 and Claim 14, we have
$$2^{n-1}\Phi(e_i) = \Phi(\xi_{1_A}(e_i, e_i)) = \xi_{1_A}(\Phi(e_i), e_i) + \xi_{1_A}(e_i, \Phi(e_i)) = 2^{n-1}(\Phi(e_i)e_i + e_i\Phi(e_i)).$$
Multiplying both sides of the above equation by e_j $(1 \le i \ne j \le 2)$ from the left and right respectively, we obtain that $e_j \Phi(e_i) e_j = 0$. Similarly, multiplying both sides of the above equation by e_i from the left and right respectively, we have that $e_i \Phi(e_i) e_i = 0$. Hence $\Phi(e_i) = e_1 \Phi(e_i) e_2 + e_2 \Phi(e_i) e_1$, $i = 1, 2$. On the other hand, we have
$$\begin{aligned}0 = \Phi(\xi_{1_A}(e_1, e_2)) &= \xi_{1_A}(\Phi(e_1), e_2) + \xi_{1_A}(e_1, \Phi(e_2))\\ &= 2^{n-2}(e_1\Phi(e_1)e_2 + e_2\Phi(e_1)e_1 + e_1\Phi(e_2)e_2 + e_2\Phi(e_2)e_1).\end{aligned}$$

Now, by the additivity of Φ, we can get that
$$\Phi(1_A) = \Phi(e_1) + \Phi(e_2) = e_1\Phi(e_1)e_2 + e_2\Phi(e_1)e_1 + e_1\Phi(e_2)e_2 + e_2\Phi(e_2)e_1 = 0.$$

Claim 16. For all $a, b \in A$, we have $\Phi(a \bullet b) = \Phi(a) \bullet b + a \bullet \Phi(b)$.

For every $a, b \in A$, it follows from $\Phi(1_A) = 0$ that
$$2^{n-2}\Phi(a \bullet b) = \Phi(\xi_{1_A}(a, b)) = \xi_{1_A}(\Phi(a), b) + \xi_{1_A}(a, \Phi(b)) = 2^{n-2}(\Phi(a) \bullet b + a \bullet \Phi(b)).$$

Hence $\Phi(a \bullet b) = \Phi(a) \bullet b + a \bullet \Phi(b)$.

Claim 17. For all $a \in A$, $\Phi(a^*) = \Phi(a)^*$.

For every $a \in A$, by Claims 12, 15, 16, we have $\Phi(a) + \Phi(a^*) = \Phi(a \bullet 1_A) = \Phi(a) \bullet 1_A = \Phi(a) + \Phi(a)^*$. Hence $\Phi(a^*) = \Phi(a)^*$.

Now let us define a map $\phi : A \to A$ by $\phi(a) = \Phi(a) - \Xi_{y,z}(a)$ for all $a \in A$, where $\Xi_{y,z}$ is ∗-inner derivation on A, with $y = e_1\Phi(e_1)e_2 + e_2\Phi(e_1)e_1$ and $z = e_1$. It is easy to see the following properties.

Claim 18.   (1) For all $a, b \in A$, $\phi(a \bullet b) = \phi(a) \bullet b + a \bullet \phi(b)$;
  (2) $\phi(e_i) = 0$, $i = 1, 2$;
  (3) ϕ is additive;
  (4) For all $a \in A$, $\phi(a^*) = \phi(a)^*$;
  (5) ϕ is an additive derivation if and only if Φ is an additive derivation.

Claim 19. $\phi(a_{ij}) \in A_{ij}$, $i,j = 1,2$.

Let $a_{ij} \in A_{ij}$, $1 \le i \ne j \le 2$. On the one hand, it follows from $\phi(e_i) = 0$ that

$$\phi(a_{ij}) = \phi(e_i \bullet a_{ij}) = e_i \bullet \phi(a_{ij}) = e_i\phi(a_{ij}) + \phi(a_{ij})e_i.$$

Hence $e_i\phi(a_{ij})e_i = e_j\phi(a_{ij})e_j = 0$. On the other hand, we have

$$0 = \phi(a_{ij} \bullet e_i) = \phi(a_{ij}) \bullet e_i = \phi(a_{ij})e_i + e_i\Phi(a_{ij})^*.$$

Hence $e_j\phi(a_{ij})e_i = 0$. Thus $\phi(a_{ij}) = e_i\phi(a_{ij})e_j \in A_{ij}$, $1 \le i \ne j \le 2$. Let $a_{ii} \in A_{ii}$, $i = 1,2$. Then

$$0 = \phi(e_j \bullet a_{ii}) = e_j \bullet \phi(a_{ii}) = e_j\phi(a_{ii}) + \phi(a_{ii})e_j.$$

Hence $e_i\Phi(a_{ii})e_j = e_j\Phi(a_{ii})e_i = e_j\Phi(a_{ii})e_j = 0$. Now we get $\Phi(a_{ii}) = e_i\Phi(a_{ii})e_i \in A_{ii}$, $i = 1,2$.

Claim 20. *Let $a_{ii}, b_{ii} \in A_{ii}$ and $a_{ij}, b_{ij} \in A_{ij}$, $1 \le i \ne j \le 2$. Then*

$$\phi(a_{ii}b_{ii}) = \phi(a_{ii})b_{ii} + a_{ii}\phi(b_{ii}), \ \ \phi(a_{ii}b_{ij}) = \phi(a_{ii})b_{ij} + a_{ii}\phi(b_{ij}),$$

$$\phi(a_{ij}b_{ji}) = \phi(a_{ij})b_{ji} + a_{ij}\phi(b_{ji}), \ \ \phi(a_{ij}b_{jj}) = \phi(a_{ij})b_{jj} + a_{ij}\phi(b_{jj}),$$

$$\phi(a_{ij}b_{ij}) = \phi(a_{ij})b_{ij} + a_{ij}\phi(b_{ij}).$$

It follows from Claim 19 that

$$\phi(a_{ii}b_{ij}) = \phi(a_{ii} \bullet b_{ij}) = \phi(a_{ii}) \bullet b_{ij} + a_{ii} \bullet \phi(b_{ij}) = \phi(a_{ii})b_{ij} + a_{ii}\phi(b_{ij}),$$

that is $\phi(a_{ii}b_{ij}) = \phi(a_{ii})b_{ij} + a_{ii}\phi(b_{ij})$. By flexibility of alternative algebras and for any $c_{ij} \in A_{ij}$, we have

$$\begin{aligned}
\phi(a_{ii}b_{ii}) \cdot c_{ij} + a_{ii}b_{ii} \cdot \phi(c_{ij}) = \phi(a_{ii}b_{ii} \cdot c_{ij}) &= \phi(a_{ii}) \cdot b_{ii}c_{ij} + a_{ii} \cdot \phi(b_{ii}c_{ij}) \\
&= \phi(a_{ii}) \cdot b_{ii}c_{ij} + a_{ii} \cdot \phi(b_{ii})c_{ij} + a_{ii} \cdot b_{ii}\phi(c_{ij}) \\
&= \phi(a_{ii})b_{ii} \cdot c_{ij} + a_{ii}\phi(b_{ii}) \cdot c_{ij} + a_{ii}b_{ii} \cdot \phi(c_{ij}).
\end{aligned}$$

Then $(\phi(a_{ii}b_{ii}) - \phi(a_{ii})b_{ii} - a_{ii}\phi(b_{ii}))c_{ij} = 0$ for any $c_{ij} \in A_{ij}$. It follows from (♠) and (♣) that $\phi(a_{ii}b_{ii}) = \phi(a_{ii})b_{ii} + a_{ii}\phi(b_{ii})$. It follows from Claim 19 that

$$\begin{aligned}
\phi(a_{ij}b_{ji}) + \phi(b_{ji}a_{ij}^*) = \phi(a_{ij} \bullet b_{ji}) &= \phi(a_{ij}) \bullet b_{ji} + a_{ij} \bullet \phi(b_{ji}) \\
&= \phi(a_{ij})b_{ji} + a_{ij}\phi(b_{ji}) + b_{ji}\phi(a_{ij})^* + \phi(b_{ji})a_{ij}^*,
\end{aligned}$$

hence $\phi(a_{ij}b_{ji}) = \phi(a_{ij})b_{ji} + a_{ij}\phi(b_{ji})$. Again by flexibility of alternative algebras and for any $c_{ji} \in A_{ji}$ $(i \ne j)$, we have

$$\begin{aligned}
c_{ji} \cdot \phi(a_{ij}b_{jj}) + \phi(c_{ji}) \cdot a_{ij}b_{jj} = \phi(c_{ji} \cdot a_{ij}b_{jj}) &= \phi(c_{ji}a_{ij} \cdot b_{jj}) \\
&= \phi(c_{ji}a_{ij})b_{jj} + c_{ji}a_{ij}\phi(b_{jj}) \\
&= \phi(c_{ji})a_{ij} \cdot b_{jj} + c_{ji}\phi(a_{ij}) \cdot b_{jj} + c_{ji}a_{ij} \cdot \phi(b_{jj}) \\
&= \phi(c_{ji}) \cdot a_{ij}b_{jj} + c_{ji} \cdot \phi(a_{ij})b_{jj} + c_{ji} \cdot a_{ij}\phi(b_{jj}).
\end{aligned}$$

Hence $c_{ji}(\phi(a_{ij}b_{jj}) - \phi(a_{ij})b_{jj} - a_{ij}\phi(b_{jj})) = 0$ for any $c_{ji} \in A_{ji}$. It follows from (♠) and (♣) that $\phi(a_{ij}b_{jj}) = \phi(a_{ij})b_{jj} + a_{ij}\phi(b_{jj})$. Since ϕ is a $*$-Jordan derivation we have

$$\begin{aligned}
\phi(a \bullet (bc + cb^*)) &= \phi(a) \bullet (bc + cb^*) + a \bullet \phi(bc + cb^*) \\
&= \phi(a)(bc + cb^*) + (bc + cb^*)\phi(a)^* + a\phi(bc + cb^*) + \phi(bc + cb^*)a^*.
\end{aligned}$$

Tanking $a = e_j$, $b = a_{ij}$ and $c = b_{ij}$ in (2) we get

$$
\begin{aligned}
\phi(a_{ij}b_{ij}) &= e_j\phi(a_{ij}b_{ij} + b_{ij}a_{ij}^*) + \phi(a_{ij}b_{ij} + b_{ij}a_{ij}^*)e_j \\
&= e_j \cdot \phi(a_{ij})b_{ij} + e_j \cdot a_{ij}\phi(b_{ij}) + e_j \cdot \phi(b_{ij})a_{ij}^* + e_j \cdot b_{ij}\phi(a_{ij}^*) \\
&= \phi(a_{ij})b_{ij} + a_{ij}\phi(b_{ij}).
\end{aligned}
$$

Claim 21. $\Phi(ab) = \phi(a)b + a\phi(b)$ for all $a, b \in A$.

Write $a = \sum_{i,j=1}^{2} a_{ij}, b = \sum_{i,j=1}^{2} b_{ij} \in A$. Then $ab = a_{11}b_{11} + a_{11}b_{12} + a_{12}b_{12} + a_{12}b_{21} + a_{12}b_{22} + a_{21}b_{11} + a_{21}b_{12} + a_{21}b_{21} + a_{22}b_{21} + a_{22}b_{22}$. It follows from Claim 20 and the additivity of ϕ that $\phi(ab) = \phi(a)b + a\phi(b)$. By the definition of ϕ, we obtain that $\Phi(ab) = \Phi(a)b + a\Phi(b)$. Now, by Claim 12, Claim 17 and Claim 20, we have proved that Φ is an additive ∗-derivation. This completes the proof.

3. Application

A complete normed alternative complex ∗-algebra A is called of alternative C^*-algebra if it satisfies the condition: $\|a^*a\| = \|a\|^2$, for all elements $a \in A$. A non-zero element $p \in A$ is called a projection if it is self-adjoint and verifies the condition $p^2 = p$. Alternative C^*-algebras are non-associative generalizations of C^*-algebras and appear in various areas in Mathematics (see more details in the references [11] and [12]). An alternative C^*-algebra A is called of alternative W^*-algebra if it is a dual Banach space and a prime alternative W^*-algebra is called alternative W^*-factor. It is well known that non-zero alternative W^*-algebras are unital and that an alternative W^*-algebra A is a factor if and only if its center is equal to $\mathbb{C}1_A$, where 1_A is the unit of A.

Theorem 22. *Let A be an alternative W^*-factor. If a map $\Phi : A \to A$ satisfies*

$$
\Phi(a_1 \bullet a_2 \bullet \cdots \bullet a_n) = \sum_{k=1}^{n} a_1 \bullet \cdots \bullet a_{k-1} \bullet \Phi(a_k) \bullet a_{k+1} \bullet \cdots \bullet a_n
$$

for all $a_{n-1}, a_n \in A$ and $a_i = 1_A$ for all $i \in \{1, 2, \ldots, n-2\}$, then Φ is an additive ∗-derivation.

Corollary 23. *Let A be an alternative W^*-factor. Then Φ is a nonlinear ∗-Jordan-type derivation on A if and only if Φ is an additive ∗-derivation.*

4. Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

References

[1] M. Brešar, M. Fošner. *On ring with involution equipped with some new product*, Publ. Math., **57**:1-2 (2000), 121–134. Zbl 0969.16013

[2] J.C. da Motta Ferreira, B.L.M. Ferreira, *Additivity of n-multiplicative maps on alternative rings*, Commun. Algebra, **44**:4 (2016), 1557–1568. Zbl 1403.17031

[3] L. Dai, F. Lu, *Nonlinear maps preserving Jordan ∗-products*, J. Math. Anal. Appl., **409**:1 (2014), 180–188. Zbl 1339.47047

[4] V. Darvish, M. Nouri, M. Razeghi, A. Taghavi, *Nonlinear ∗-Jordan triple derivations on prime ∗-algebras*, Rocky Mt. J. Math., **50**:2 (2020) 543–549. Zbl 1435.16011

[5] B.L.M. Ferreira, *Additivity of elementary maps on alternative rings*, Algebra Discrete Math., **28**:1 (2019), 94–106. Zbl 7268044

[6] B.L.M. Ferreira, B.T. Costa, *-Jordan-type maps on C^*-algebras, Commun. Algebra, **49**:12 (2021), 5073–5082. Zbl 7431283

[7] J.C. da Motta Ferreira, B.L.M. Ferreira, Additivity of n-multiplicative maps on alternative rings, Commun. Algebra, **44**:4 (2016), 1557–1568. Zbl 1403.17031

[8] B.L.M. Ferreira, H. Guzzo, R.N. Ferreira, An approach between the multiplicative and additive structure of a Jordan ring, Bull. Iran. Math. Soc., **47**:4 (2021), 961–975. Zbl 1472.17002

[9] B.L.M. Ferreira, H. Guzzo, Lie maps on alternative rings, Boll. Unione Mat. Ital., **13**:2 (2020), 181–192. Zbl 1434.17007

[10] B.L.M. Ferreira, H. Guzzo, I. Kaygorodov, Lie maps on alternative rings preserving idempotents, Colloq. Math., **166**:2 (2021), 227–238. Zbl 7472578

[11] M.C. García, Á.R. Palacios, Non-associative normed algebras. Vol. 1. The Vidav-Palmer and Gelfand-Naimark theorems, in Encyclopedia of Mathematics and its Applications, **154**, Cambridge University Press, Cambridge, 2014. Zbl 1322.46003

[12] M.C. García , Á.R. Palacios, Non-associative normed algebras. Vol. 2. Representation theory and the Zel'manov approach, in Encyclopedia of Mathematics and its Applications, **167**, Cambridge University Press, Cambridge, 2018. Zbl 1390.17001

[13] I. Hentzel, E. Kleinfeld, H. Smith, Alternative rings with idempotent, J. Algebra, **64** (1980), 325–335. Zbl 0441.17010

[14] D. Huo, B. Zheng, H. Liu, Nonlinear maps preserving Jordan triple η-*-products, J. Math. Anal. Appl., **430**:2 (2015), 830–844. Zbl 1331.47054

[15] C. Li, F. Lu, X. Fang, Nonlinear $\{\xi\}$-Jordan *-derivations on von Neumann algebras, Linear Multilinear Algebra, **62**:4 (2014) 466–473. Zbl 1293.47037

[16] C. Li, F. Lu, X. Fang, Nonlinear mappings preserving product $XY + YX^*$ on factor von Neumann algebras, Linear Algebra Appl., **438**:5 (2013), 2339–2345. Zbl 1276.47047

[17] C. Li, F. Lu, T. Wang, Nonlinear maps preserving the Jordan triple *-product on von Neumann algebras, Ann. Funct. Anal., **7**:3 (2016), 496–507. Zbl 1351.47028

[18] C. Li, F. Lu, Nonlinear maps preserving the Jordan triple 1-*-product on von Neumann algebras, Complex Anal. Oper. Theory, **11**:1 (2017), 109–117. Zbl 1454.47043

[19] L. Molnár, A condition for a subspace of $B(H)$ to be an ideal, Linear Algebra Appl., **235** (1996) 229–234. Zbl 0852.46021

[20] P. Šemrl, Quadratic functionals and Jordan *-derivations, Stud. Math., **97**:3 (1991) 157–165. Zbl 0761.46047

[21] P. Šemrl, On Jordan *-derivations and an application, Colloq. Math., **59**:2 (1990) 241–251. Zbl 0723.46044

[22] A. Taghavi, H. Rohi, V. Darvish, Non-linear *-Jordan derivations on von Neumann algebras, Linear Multilinear Algebra, **64**:3 (2016), 426–439. Zbl 1353.46049

[23] F. Zhang, Nonlinear skew Jordan derivable maps on factor von Neumann algebras, Linear Multilinear Algebra, **64**:10 (2016) 2090–2103. Zbl 1353.47073

[24] F. Zhao, C. Li, Nonlinear *-Jordan triple derivations on von Neumann algebras, Math. Slovaca, **68**:1 (2018) 163–170. Zbl 6842060

[25] F. Zhao, C. Li, Nonlinear maps preserving the Jordan triple *-product between factors, Indag. Math., New Ser., **29**:2 (2018) 619–627. Zbl 1425.46050

ALINE JAQUELINE DE OLIVEIRA ANDRADE
FEDERAL UNIVERSITY OF ABC,
5001, DOS ESTADOS AVE.,
SANTO ANDRÉ, 09210-580, BRAZIL
Email address: aline.jaqueline@ufabc.edu.br

GABRIELA COTRIM MORAES
FEDERAL UNIVERSITY OF ABC,
5001, DOS ESTADOS AVE.,
SANTO ANDRE, 09210-580, BRAZIL
Email address: gcotrim@gmail.com

Ruth Nascimento Ferreira
Federal University of Technology,
800, Professora Laura Pacheco Bastos ave.,
Guarapuava, 85053-510, Brazil
Email address: ruthnascimento@utfpr.edu.br

Bruno Leonardo Macedo Ferreira
Federal University of Technology,
800, Professora Laura Pacheco Bastos ave.,
Guarapuava, 85053-510, Brazil
Email address: brunolmfalg@gmail.com