Review of the manuscript of
M. Fraiman and E. Troitsky
"Reidemeister classes in some wreath products"

Let G be a group and φ be an endomorphism of G. Elements x, y from G are said to be φ-conjugated if there exists an element $z \in G$ such that $x = zy\varphi(z)^{-1}$. The relation of φ-conjugation is an equivalence relation and it divides G into φ-conjugacy classes. The number $R(\varphi)$ of these classes is called the Reidemeister number of φ. The Reidemeister number is either an integer or the infinity. If $R(\varphi) = \infty$ for all $\varphi \in \mathrm{Aut}(G)$, then G is said to possess the R_∞-property.

Twisted conjugacy classes appear naturally in group theory (Dekimpe, Goncalves, Nasybullov, Bardakov), knot theory (Makanin), Nielsen-Reidemester fixed point theory (Felshtyn, Dekimpe, Goncalves, Wong, Lee), group based cryptography (Shpilrain, Ushakov, Roman'kov). The study of groups having (or not) the R_∞ property was initiated in the mid 90s and is now a very active research topic.

The authors of the paper under review study the Reidemeister numbers for automorphisms of wreath products $G \wr \mathbb{Z}^k$, where G is a finite abelian group. They find quite general sufficient conditions under which $G \wr \mathbb{Z}^k$ doesn't possess the R_∞-property. Moreover, they prove that for finite order automorphisms of $G \wr \mathbb{Z}^k$ with $R(\varphi) < \infty$ the finitely dimensional twisted Burnside-Frobenius theorem holds.

This work can be considered a natural continuation of the work [Internat. J. Algebra Comput., V. 16, N. 5, 2006, 875-886] by D. Goncalves and P. Wong and the work [Sib. Electronic Math. Rep., V. 17, 2020, 890-898] by M. Fraiman. The work is of interest to specialists in the theory of twisted conjugacy classes, and to specialists in the Nielsen-Reidemeister fiex-point theory. The results of the work improve the known results from the articles cited above. I recommend that the article be accepted for publication in Siberian Electronic Mathematical Reports after correcting some deficiencies in the text. The list of comments and suggestions is given below.

1. Title of the manuscript: I suggest to change the title of the work since the title "Reidemeister classes in some wreath products" is too general. It doesn't reflect what is really done in the paper. I suggest the title "Reidemeister classes in wreath products of abelian groups"

2. Throughout the paper: Change "Abelian group" by "abelian group".

3. Throughout the paper: The names of areas of mathematics do not need to be capitalized. In particular, it is necessary to write "topological dynamics" instead of "Topological Dynamics" and so on.

4. The seond paragraph of the abstract is quite difficult to read. I suggest to rewrite it in the following form: Moreover, we prove that if φ is a finite order automorphism of $G \wr \mathbb{Z}^k$ with $R(\varphi) < \infty$, then $R(\varphi)$ is equal to the number of fixed points of the map $[\rho] \mapsto [\rho \circ \varphi]$ defined on the set of equivalence classes of finite dimensional irreducible unitary representations of $G \wr \mathbb{Z}^k$.

5. Page 2, line 3: Write "[1,3,7,12,21,23,26,27]" instead of "[7,21,1,3,12,27,26,21]"

6. Page 2, paragraph 4: The authors write that the study of twisted conjugacy classes in $G \wr \mathbb{Z}^k$ is more difficult than in $G \wr \mathbb{Z}$ since $\mathbb{Z}^k$ has more automorphisms than $\mathbb{Z}$. It is necessary to explain here why the number of automorphisms of $\mathbb{Z}^k$ plays a role in the study of twisted conjugacy classes in $G \wr \mathbb{Z}^k$.

7. Throughout the paper: The authors denote by $C(\varphi)$ the set $\{g \in G \mid \varphi(g) = g\}$. This set is usually denoted by $\mathrm{Fix}(\varphi)$. I suggest to use the classical notation $\mathrm{Fix}(\varphi)$ instead of $C(\varphi)$.

8. Page 2, definition 2.1: Change "Denote $C(\varphi)$" by "Denote by $C(\varphi)$".

9. Page 2, theorem 2.2, item 4: Write "$g_1, \ldots, g_R$" insted of "$g_1, \ldots g_R$" (missed coma).
10. Page 3, line right after the commutative diagram: add "," before "then".
11. Page 5, case 2, line 6: Change "corresponding Reidemeister number = $\det(E - M) = 3$" by "corresponding Reidemeister number is equal to $\det(E - M) = 3$".
12. Page 6, the last two paragraphs: In general it is clear what the authors mean speaking about $\mathbb{Z}_{p^i}$. However, in the formulation of Theorem 3.1 the group G was written in the form $G = \oplus_i (\mathbb{Z}_{p_i^{r_i}})^{d_i}$. So, I suggest to write "$\mathbb{Z}_{p^r}$" instead of "$\mathbb{Z}_{p^i}$".
13. Page 7, Theorem 4.1: I suggest to reformulate the theorem in the following way. Let G be a finite abelian group, and $\varphi \in \mathrm{Aut}(G \wr \mathbb{Z}^k)$ be a finite order automorphism. Then $R(\varphi') \in \{1, \infty\}$.
14. Page 7, Corollary 4.2: I suggest to write the formulation of the Corollary in details: Let G be a finite abelian group, and $\varphi \in \mathrm{Aut}(G \wr \mathbb{Z}^k)$ be an automorphism of finite order. Then φ has the $TBFT_f$ property.
15. Page 7, list of references: Format the reference list in a uniform way.