Referee Report

Paper: Recognition of the group $E_6(5)$ by prime graph
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Given a finite group G, the prime graph $\Gamma(G)$ is constructed as follows: the vertex set is the set pf prime divisors of $|G|$ and two distinct primes p and q are joined by an edge if and only if G has an element of order pq. A group G is said to be recognizable by prime graph if every finite group H with $\Gamma(H) = \Gamma(G)$ is isomorphic to G.

The main result of the paper is that the exceptional group $E_6(5)$ is recognizable by prime graph. This result is new, however, its proof is very similar to the proof of one of the previously known results. Namely, Step 1 and Steps 3-6 of the present paper are slight modifications of Lemmas 3.1 and 3.4–3.7 in [15], where $E_6(3)$ and ${}^2E_6(3)$ were proved to be recognizable by prime graph. So I think that the paper is not interesting enough to be published in 'Siberian Electronic Mathematical Reports'.

Also I found some misprints and inaccuracies.

Page 2: Lemma 2.1 was proved not in [24] but in [A.V. Vasil'ev, On connection between the structure of finite group and the properties of its prime graph, Sib. Math. J., 46:3 (2005), 396–404].

Page 2, Remark 3.1: The equality $\rho(2, E_6(5)) = \{2, 19, 601, 829\}$ contradicts the previous equality $t(2, E_6(5)) = 3$ and the fact that, by definition, $t(2, G) = |\rho(2, G)|$.

Page 3, Step 2: By [27], the fact that 829 is the largest prime in $\in \pi(S)$ implies that S is either of one of the groups in the corresponding row of Table 3 in [27] (non-generic groups) or one of the generic groups defined on page 2 of [27]. So in Step 2, S can also be one of the groups $L_2(829)$, A_{829}, $\ldots$, $A_{p'}$, where p' is the smallest prime greater than 829.

Page 3, Step 3: The authors write that 'By [1, Theorem 19.9], $C_L(\gamma) = F_4(5)$'. But [1] is about involutions in Chevalley groups over fields of even order, so it is not applicable to $E_6(5)$. Also there is a misprint in the next sentence: it should be '$601 \in \pi(F_4(5))$' instead of '$601 \in \pi(F_4(3))$'.

Page 3, Step 4: I do not understand the reference to [13] in the following: 'Since K is nilpotent, by [13] K is an elementary abelian p-group'.

Page 3, Step 5: It should be $O_8^+(5)$ instead of $O_8(5)$.

Page 3, Step 5: The reference to Gorenstein's book [3] in 'By [3], P is cyclic' is too general.

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