

REFeree's REPORT

on the article '*Homogenized acoustic equations for a layered medium consisting of a viscoelastic material and a viscous compressible fluid*'

(author: V. V. Shumilova)

submitted for publication in *Siberian Electronic Mathematical Reports*
(«Сибирские электронные математические известия»)

The article written by V. V. Shumilova is devoted to the qualitative analysis of the homogenized model of joint motion of a viscoelastic medium and a viscous compressible fluid, where the two media are situated as periodically alternating layers. The model was derived in the previous works of the author jointly with A. S. Shamaev [7-9]. In the present article, the author succeeds to solve the cell problems explicitly, which means that the complete information is revealed from the microstructure. Then, the obtained solutions are inserted into the previously obtained formulas for effective homogenized coefficients. Thus, the effective macroscopic description of the homogenized continuum is completely derived from the microstructure. The form of the resulting formulas allows one to make an interesting conclusion, which is that the effective coefficients do not depend on the exact relative position of the layers in the pattern cell of periodicity set at the microscopic level, but depend only on the volume fractions of two continuums in this cell. This conclusion is formulated as Theorem 1 and declared as the main result of the work. In the end of the paper, some particular and degenerate cases are considered, and the result of Theorem 1 is also confirmed for these cases.

The result of the article is new and correct. It looks very interesting in the theory of homogenization and serves as a useful continuation and addition to this theory.

There are two minor imperfection found while reading the text:

1. I strongly recommend to add the explicit formula for convolution $*$ with respect to t . It is important in order to avoid ambiguity, since the convolution in the article is fulfilled not over the whole semi-axis \mathbb{R}_t but depends on the upper limit of integration.
2. Letter h seems to be a bit overloaded, since it stands for a total volume fraction of the fluid and at the same time for the index for components of tensors. It is desirable to change notation in one of the cases.

Overall opinion on the article '*Homogenized acoustic equations for a layered medium consisting of a viscoelastic material and a viscous compressible fluid*' written and submitted by V. V. Shumilova is positive. **I recommend the article for publication** in *Siberian Electronic Mathematical Reports* («Сибирские электронные математические известия») after a mentioned minor correction.

Referee