Review on the paper of V.M. Leontiev “On the collection process for positive words”

The text is much better now, but there are a couple of misprints and one problem with Example 4.7. I will start from the latter.

Example 4.7. In this example the predicate $E^{(\lambda_1,\lambda_2,\lambda_3)}_{[[a_2,a_1],a_2]}$ is expressed through the “elementary” predicates. The first step using the formula (12) is clear:

$$E^{(\lambda_1,\lambda_2,\lambda_3)}_{[[a_2,a_1],a_2]} = P^{(\lambda_1,\lambda_2,\lambda_3)}_{[a_2,a_1],a_2} = \dots$$

The second step uses the formula (13a) with $u = 1$, $v = 0$, $Q_1 = Q_2 = a_2$ and $R_1 = a_1$, to obtain

$$\dots = E^{(\lambda_1,\lambda_2)}_{[a_2,a_1]} E^{(\lambda_3)}_{a_2} F.$$

In the text it is written that $F = P^{(\lambda_1,\lambda_3)}_{a_2,a_2} \vee (\lambda_2 = \lambda_3)$, and I do not quite understand why this is so. Consider the formula for F in Theorem 4.6:

$$F = P^{\Lambda^1_0 \Lambda^2_0}_{Q_1,Q_2} \vee (\Lambda^1_0 = \Lambda^2_0) \left((u < v) \bigwedge_{k=1}^{u} (\dots) \vee \bigvee_{k=1}^{\min\{u,v\}} (\dots) \right).$$

We have $\Lambda^1_0 = \lambda_1$, $\Lambda^2_0 = \lambda_3$, therefore after substitution we get

$$F = P^{(\lambda_1,\lambda_3)}_{a_2,a_2} \vee (\lambda_1 = \lambda_3) \left((1 < 0) \bigwedge_{k=1}^{1} (\dots) \vee \bigvee_{k=1}^{0} (\dots) \right).$$

Clearly $1 < 0$ is false and an empty disjunction $\bigvee_{k=1}^{0}(\dots)$ is also false, so after simplification

$$F = P^{(\lambda_1,\lambda_3)}_{a_2,a_2}.$$

Unless I am missing something, the formula for the original predicate should then be

$$E^{(\lambda_1,\lambda_2,\lambda_3)}_{[[a_2,a_1],a_2]} = P^{(\lambda_1,\lambda_2)}_{a_2,a_1} E^{(\lambda_3)}_{a_2} P^{(\lambda_1,\lambda_3)}_{a_2,a_2} = (\lambda_1 < \lambda_2) \wedge (\lambda_1 < \lambda_3).$$

Misprints. On page 9, Theorem 4.4: “If we collect R’s …”. It is better to avoid pluralising mathematical symbols, so it’s better to write “If we collect R …” or if you want to be precise “If we collect all occurrences of R …”.

Page 14, fourth paragraph from the top: “We say that a tuple of elements $(\lambda_1^*, \dots, \lambda_r^*) \in N$ satisfy …”. Of course, “satisfies” is better.