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DETERMINATION OF NON-STATIONARY ADSORPTION COEFFICIENT ANALYTICAL IN PART OF SPATIAL VARIABLES

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In this study the multidimensional adsorption coefficient inverse problem for the second order hyperbolic equation are considered. It is supposed that this coefficient is continuous with respect to the variables t, x and analytic in the other spatial variables. For solving this equations the scale method of Banach spaces of analytic functions is applied. The problem are reduced to a system of nonlinear Volterra integral equations and the local existence, global uniqueness, stability estimates are established.

Key words: *inverse problem, fundamental solution, local solvability, Banach space, equivalent norm.*
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1 Introduction. Formulation of problem

According to the terminology of [1], an inverse problem is called one-dimensional if the sought function depends on one variable, and multidimensional if it depends on two and more variables. For multidimensional inverse problems there are only special cases for which solvability is established. One of such classes of functions in which local solvability takes place is the class of analytic functions. The technique used here is based on the scale method of Banach spaces of analytic functions, developed in the works of L.V. Ovsyannikov [2,3] and L. Nirenberg [4]. This method was first applied to the problem of solvability of multidimensional inverse problems by V.G. Romanov [1,5,6]. The approach used in this works can be successfully applied to the study of solvability of the problem of determining the non-stationary coefficient for the first time derivative of the equation with string vibration operator in a principle part.

In this paper we study the inverse problem of determining the coefficient $a(t, x, y)$, $t \in \mathbb{R}$, $(x, y) = (x, y_1, \cdots, y_m) \in \mathbb{R}^{1+m}$, for the second order hyperbolic equation. The problem is investigated in the class of coefficients that are continuous with respect to the variables t, x and analytic in the variable y.

In [7–13] this method was used to study of the multidimensional inverse problems of determining the convolutional kernel in parabolic and hyperbolic integro-differential equations of the second order; theorems of local unique solvability of inverse problems in the class of functions with finite smoothness with respect to the time variable and analytic with respect to spatial variables. This article generalizes the results of work [1] (sec. 3) for the case of non-stationary potential.

Consider the problem of determining a pair of functions u and a satisfying the equations:

$$u_{tt} - u_{xx} - \triangle u - a(t,x,y)u_t = g(y)\delta(x)\delta(t-t_0),\ (t,x,y) \in \mathbb{R}^{2+m},\ t_0 > 0, \tag{1}$$

$$u\Big|_{t<0} \equiv 0, \tag{2}$$

in which $\triangle$ is the Laplace operator on variables $(y_1, \dots, y_m) = y$, $\delta(\cdot)$ is the Dirac delta function, $\delta'(\cdot)$ is the derivative of the Dirac delta function, t_0 is a problem parameter, therefore $u = u(t,x,y,t_0)$, $g(y)$ is a given smooth function so that $g(y) \neq 0$ for $y \in \mathbb{R}^m$.

In inverse problem it is required to find the potential $a(x,t,y)$ in (1), if the solution of the problem (1)–(3) is known for $x = 0$, i.e. the condition

$$u(t,0,y,t_0) = f(t,y,t_0),\ t > 0,\ t_0 > 0 \tag{3}$$

is given.

Following the monograph [4, sec. 3], we introduce into consideration the Banach space $A_s(r)$, $s > 0$, of functions $\varphi(y)$, $y \in \mathbb{R}^m$, which are analytic in the neighborhood of the origin and satisfy the relation

$$\|\varphi\|_s(r) := \sup_{|y|<r} \sum_{|\alpha|=0}^{\infty} \frac{s^{|\alpha|}}{\alpha!} |D^\alpha \varphi(y)| < \infty.$$

Here $r > 0,\ s > 0$ and

$$D^\alpha := \frac{\partial^{|\alpha|}}{\partial y_1^{\alpha_1} ... \partial y_m^{\alpha_m}},\ \alpha := (\alpha_1, ..., \alpha_m),$$

$$|\alpha| := \alpha_1 + ... + \alpha_m,\ \alpha! := (\alpha_1)! \dots (\alpha_m)!.$$

In the following, the parameter r will be considered fixed, while a parameter s is variable. Then, it is formed a scale of Banach spaces $A_s(r)$, $s > 0$ of analytic functions. The following property is obvious: if $\varphi(y) \in A_s(r)$, then $\varphi(y) \in A_{s'}(r)$ for all $s' \in (0,\ s)$, consequently, $A_s(r) \subset A_{s'}(r)$, if $s' \in (0,\ s)$ and the following inequality is valid:

$$\|D^\alpha \varphi\|_{s'} \leq c_\alpha \frac{\|\varphi\|_s(r)}{(s-s')^{|\alpha|}}$$

for any α with constant c_α, which depends only on α.

We present the solution of the problem (1), (2) in the form

$$u(t,x,y,t_0) = \frac{1}{2} g(y)\theta(t - t_0 - |x|) + v(t,x,y,t_0),$$

where $v(t,x,y,t_0)$ – regular function. n Substituting this expression into (1), we obtain the following problem for determining function v:

$$v_{tt} - v_{xx} = \triangle v + \frac{1}{2}\triangle g(y)\theta(t - t_0 - |x|)$$

$$+a(t,x,y)\left[\frac{1}{2} g(y)\delta(t - t_0 - |x|) + v_t(t,x,y,t_0)\right],\ (t,x,y) \in \mathbb{R}^{2+m},\ t_0 > 0, \tag{4}$$

$$v\ |_{t<0} \equiv 0. \tag{5}$$

In the next section we will replace the inverse problem (4), (5) and (2) by the equivalent integro-differential equations. In what follows, we assume that the function a is even in x.

2 Reducing the problem to the integrodifferential equations

According to the d'Alembert formula the solution of problem (4)–(5) satisfies the integral equation

$$v(t,x,y,t_0)=$$

$$=\frac{1}{2}\int\int_{\Delta(t,x)}\{\triangle v(\tau,\xi,y,t_0)+\frac{1}{2}\triangle g(y)\theta(\tau-t_0-|\xi|)$$

$$+a(\tau,\xi,y)\left[\frac{1}{2}g(y)\delta(\tau-t_0-|\xi|)+v_t(\tau,\xi,y,t_0)\right]\}d\xi d\tau,\ (t,x,y)\in\mathbb{R}^{2+m},\ t_0>0, \tag{6}$$

in which

$$\Delta(t,x)=\{(\tau,\xi)|\ 0\le\tau\le t-|x-\xi|,\ x-t\le\xi\le x+t\}.$$

Let

$$Q_T:=\{(t,t_0)|\ 0\le t_0\le t\le T\},\ T>0,$$

$$\Omega_T:=\{(t,x)|\ 0\le|x|\le t\le T-|x|\},$$

$$\Upsilon_T:=\left\{(t,x,t_0)\Big|\ |x|+t_0\le t\le T-|x|,\ 0\le t_0\le t\le T\right\}.$$

The domain Υ_T in the space of the variables x, t, t_0 is the pyramid with the base Ω_T and vertex $(0,T,T)$.

It follows from (6) that the function $v(t,x,y,t_0)$ satisfies the integral equation

$$v(t,x,y,t_0)=\frac{\triangle g(y)}{4}\tilde{S}(t,x,t_0)+\frac{g(y)}{4}\int_{\frac{x-(t-t_0)}{2}}^{\frac{x+(t-t_0)}{2}}a(t_0+|\xi|,\xi,y)d\xi+$$

$$+\frac{1}{2}\int\int_{\square(t,x,t_0)}\left[\triangle v(\tau,\xi,y,t_0)+a(\tau,\xi,y)v_t(\tau,\xi,y,t_0)\right]d\tau d\xi,\ (t,x,t_0)\in\Upsilon_T,\ y\in\mathbb{R}^m, \tag{7}$$

where $\theta(t)=1,\ t\ge0,\ \theta(t)=0,\ t<0,\ \tilde{S}(t,x,t_0)$ is area of a rectangle $\square(x,t,t_0)$ in the plane of the variables (τ,ξ) for each fixed t_0, formed by characteristics passing through the points $(0,t_0)$ and (x,t) of the differential operator $\partial^2/\partial t^2-\partial^2/\partial x^2$:

$$\square(x,t,t_0):=\left\{(\xi,\tau)\Big|\ |\xi|+t_0<\tau<t-|x-\xi|,\right.$$

$$\left.\frac{x-(t-t_0)}{2}<\xi<\frac{x+t-t_0}{2},\ 0<t_0<t\right\}.$$

Obviously, the equalities $f(t,y,t_0)=u(t,0,y,t_0)=v(t,0,y,t_0),\ t>t_0$ are true. Besides, $f(t_0+0,y,t_0)=v(t,0,y,t_0)\big|_{t=t_0+0}=0$.

First note that if $a(t,-x,y)=a(t,x,y)$, then $v(t,-x,y,t_0)=v(t,x,y,t_0)$. Calculating the derivative of the both sides of equation (7) with respect to t, we obtain

$$v_t(t,x,y,t_0) = \frac{\triangle g(y)}{2}\tilde{S}'_t(t,x,t_0)$$

$$+\frac{g(y)}{8}\left[a\left(\frac{x+t+t_0}{2},\frac{x+t-t_0}{2},y\right)+a\left(\frac{-x+t+t_0}{2},\frac{x-t+t_0}{2},y\right)\right]$$

$$+\frac{1}{2}\int_{\frac{x-(t-t_0)}{2}}^{\frac{x+(t-t_0)}{2}}\left[\triangle v\left(t-|x-\xi|,\xi,y,t_0\right)+a\left(t-|x-\xi|,\xi,y\right)v_t\left(t-|x-\xi|,\xi,y,t_0\right)\right]d\xi,$$

$$(t,x,t_0)\in\Upsilon_T,\ y\in\mathbb{R}^m.$$

Setting in this relation $x=0$ and using evenness of the functions $a(t,x,y)$, $v(t,x,y,t_0)$ in x, we get the equality:

$$f_t(t,y,t_0)=\frac{\triangle g(y)}{2}\tilde{S}'_t(t,0,t_0)+\frac{g(y)}{4}a\left(\frac{t-t_0}{2},\frac{t+t_0}{2},y\right)$$

$$+\int_0^{\frac{t-t_0}{2}}\left[\triangle v\left(t-\xi,\xi,y,t_0\right)+a\left(t-\xi,\xi,y\right)v\left(t-\xi,\xi,y,t_0\right)\right]d\xi,\ (t,t_0)\in Q_T,\ y\in\mathbb{R}^m.$$

Rewrite this equation substituting $(t-t_0)/2$ by $|x|$, $(t+t_0)/2$ - by t and solving with respect $a(t,x,y)$. Then we have

$$a(t,x,y)=\frac{2\triangle g(y)}{g(y)}\tilde{S}'_t(t,0,t-2|x|)+\frac{4}{g(y)}f_t\left(t+|x|,y,t-|x|\right)$$
$$-\frac{4}{g(y)}\int_0^{|x|}\Bigg[\triangle v\left(t+|x|-\xi,\xi,y,t-|x|\right)$$
$$+a\left(t+|x|-\xi,\xi,y\right)v_t\left(t+|x|-\xi,\xi,y,t-|x|\right)\Bigg]d\xi,\ (t,x)\in\Omega_T,\ y\in\mathbb{R}^m. \tag{8}$$

Thus, in order to find the value of the function a in the point (t,x,y), it is necessary to integrate function $a(t,x,y)$ itself by segment with ends $(t+|x|,0,\ y,\ 0)$, $(t,|x|,\ y,\ 0)$, and $v(t,x,y,t_0)$ — by segment with ends $(t+|x|,0,y,\ t-|x|)$, $(\ t,|x|,y,\ t-|x|)$, which belong to the domain $\Upsilon_T\times\mathbb{R}^m$.

Note that the function v, even with respect to $x=0$, satisfies the condition $\partial v/\partial x|_{x=0}$. Taking into account this fact and considering the equations (4), (5), (3) for v in the domain $x>0$ we obtain

$$\frac{\partial^2 v}{\partial t^2}-\frac{\partial^2 v}{\partial x^2}-\triangle v-a(t,x,y)v_t=0,\ 0<x<t-t_0,\ y\in\mathbb{R}^m,$$

$$v\Big|_{x=0}=f(t,y,t_0),\ \left.\frac{\partial v}{\partial x}\right|_{x=0}=0,\ 0<t-t_0\le T,\ y\in\mathbb{R}^m,$$

then in accordance with the d'Alembert formula, which gives the Cauchy problem solution with an initial data on $x=0$, we find:

$$v(t,x,y,t_0)=v_0(t,x,y,t_0)+\frac{1}{2}\iint\limits_{\triangle'(t,x)}\Bigg[\triangle v\left(\tau,\xi,y,t_0\right)+a\left(\tau,\xi,y\right)v_t\left(\tau,\xi,y,t_0\right)\Bigg]d\xi d\tau, \tag{9}$$

where

$$v_0(t,x,y,t_0) = \frac{1}{2}\left[f(t+x,y,t_0) + f(t-x,y,t_0)\right],$$

$$\triangle'(t,x) := \{(\tau,\xi) \mid 0 < \xi < x, |\tau - t| < x - \xi\}, \; 0 < x < t - t_0 < T - x, \; y \in \mathbb{R}^m.$$

Considering (8) for $x \geq 0$, we have:

$$a(t,x,y) = a_0(t,x,y)$$

$$-\frac{4}{g(y)} \int_0^x \left[\triangle v\,(t+x-\xi,\xi,y,t-x) + a(t+x-\xi,\xi,y) v_t(t+x-\xi,\xi,y,t-x)\right] d\xi, \quad (10)$$

where

$$a_0(t,x,y) = \frac{4}{g(y)} f_t(t+x,y,t-x) + \frac{2\triangle g(y)}{g(y)} \tilde{S}'_t(t,0,t-2x), \; 0 \leq x \leq t \leq T - x, \; y \in \mathbb{R}^m.$$

To close the system of equations (9), (10), we use

$$v_t(t,x,y,t_0) = v_{0t}(t,x,y,t_0) +$$

$$\frac{1}{2} \int_0^x \Big[\triangle v\,(t-x+\xi,\xi,y,t_0) - \triangle v\,(t+x-\xi,\xi,y,t_0)$$

$$+ a\,(t-x+\xi,\xi,y)\, v_t\,(t-x+\xi,\xi,y,t_0) - a\,(t+x-\xi,\xi,y)\, v_t\,(t+x-\xi,\xi,y,t_0)\Big] d\xi d\tau, \quad (11)$$

The system of equations (9), (10) is a closed integro-differential equations for the functions a, v, v_t. Note that the operator $\triangle$ in the function v appears in the system only under the integral signs.

Let $w := v_t$ and $w_0 := v_{0t}$. We will henceforth consider system (9)–(11) in the domain

$$D_T = \Upsilon'_T \times \mathbb{R}^m, \; \Upsilon'_T = \{(t,x,t_0) \mid 0 \leq x + t_0 \leq t \leq T - x\}.$$

3 The Main Results and Its Proof

Let $C_{(t,x,t_0)}(\Upsilon'_T;\, A_{s_0})$ denote the class of functions with values in A_{s_0} ($s_0 > 0$) which are continuous in the variables (t,x,t_0) in the domain Υ'_T. For fixed (t,x,t_0), the norm of a function $v(t,x,y,t_0)$ in A_{s_0} will be denoted by $\|v\|_{s_0}(t,x,t_0)$. The norm of a function v in $C_{(t,x,t_0)}(\Upsilon'_T;\, A_{s_0})$ is defined by the equality

$$\|v\|_{C_{(t,x,t_0)}(\Upsilon'_T;\, A_{s_0})} = \sup_{(t,x,t_0)\in\Upsilon'_T} \|v\|_{s_0}(t,x,t_0).$$

Let $C_{(t,x)}(G_T;\, A_{s_0})$ be the class of functions with values in A_{s_0} which are continuous in the variables (t,x) in the domain $G_T = \{(t,x) \mid 0 \leq x \leq t \leq T - x\}$. For fixed (t,x) the norm of a function $a(t,x,y)$ in A_{s_0} will be denoted by $\|a\|_{s_0}(t,x)$. The norm of a function a in $C_{(t,x)}\left(G_T;\ A_{s_0}\right)$ is defined as

$$\|a\|_{C_{(t,x)}\left(G_T;\,A_{s_0}\right)} = \sup_{(t,x)\in G_T} \|a\|_{s_0}(t,x).$$

Denote also by $C\left(Q_T; A_{s_0}\right)$ the class of functions with values in A_{s_0} which are continuous with respect to t, t_0 in domain Q_T.

Theorem 1. *Let $f\left(+t_0, y, t_0\right) = 0$, $|g(x)| \geq g_0 > 0$, g_0 is a known number, and*

$$\left\{\frac{1}{g(y)}, \frac{\triangle g(y)}{g(y)}\right\} \in A_{s_0};\ \left\{f(t,y,t_0),\ f_t(t,y,t_0)\right\} \in C\left(Q_T; A_{s_0}\right),$$

in addition, the relations

$$\max\left\{2\left\|\frac{\triangle g(y)}{g(y)}\right\|_{s_0}, \max_{(t,t_0)\in Q_T}\|f(t,y,t_0)\|_{s_0}, \max_{(t,t_0)\in Q_T}\left\|\frac{4f_t(t,y,t_0)}{g(y)}\right\|_{s_0}\right\} \leq \frac{R}{2}$$

are valid for some fixed $s_0 > 0$, R.

Then there there is such a number $b \in \left(0, T/(2s_0)\right)$, $b = b\left(s_0, R, T\right)$ that for each $s \in (0, s_0)$ in the domain $D_T \cap \left\{(t,x,y,t_0) : 0 \leq x + t_0 \leq b(s_0 - s)\right\}$ there exists the unique solution to the equations (9)–(111) and $v, w \in C_{(t,x,t_0)}\left(P_{sT};\ A_{s_0}\right)$, $a(t,x,y) \in C_{(t,x)}\left(K_{sT};\ A_{s_0}\right)$, where $P_{sT} = \Upsilon'_T \cap \left\{(t,x,t_0) : 0 \leq x + t_0 < b(s_0 - s)\right\}$, $K_{sT} = G_T \cap \left\{(t,x,t_0) : 0 \leq x + t_0 < b(s_0 - s)\right\}$, moreover

$$\left\|v - v_0\right\|_s(t,x,t_0) \leq R,\ (t,x,t_0) \in P_{sT},$$

$$\left\|w - w_0\right\|_s(t,x) \leq \frac{R}{s_0 - s}, \quad \left\|a - a_0\right\|_s(t,x) \leq \frac{R}{s_0 - s},\ (t,x) \in K_{sT}.$$

Proof. Under the conditions of Theorem 1 we have

$$v_0, w_0 \in C_{(t,x,t_0)}\left(\Upsilon'_T;\ A_{s_0}\right),\ a_0 \in C_{(t,x)}\left(G_T;\ A_{s_0}\right),$$

$$\|v_0\|_s(t,x,t_0) \leq R,\ \|w_0\|_s(t,x,t_0) \leq R,\ (t,x,t_0) \in \Upsilon'_T,$$

$$\|a_0\|_s(t,x) \leq R,\ (t,x) \in G_T,\ 0 < s < s_0.$$

Let b_n denote the members of the monotone decreasing sequence that is defined by the equalities

$$b_{n+1} = \frac{b_n}{1 + 1/(n+1)^2},\ n = 0,\ 1,\ 2,\ \ldots.$$

Denote

$$b = \lim_{n\to\infty} b_n = b_0 \prod_{n=0}^{\infty}\left(1 + 1/(n+1)^2\right)^{-1}.$$

The number $b_0 \in (0, T/(2s_0))$ will be chosen in an appropriate way. For the system of equations (9)–(11) we construct the process of successive approximations according to the following scheme:

$$v_{n+1}(t,x,y,t_0)=v_0(t,x,y,t_0)+$$

$$+\frac{1}{2}\iint\limits_{\triangle'(t,x)}\left[\triangle v_n\left(\tau,\xi,y,t_0\right)+a_n\left(\tau,\xi,y\right)v_n\left(\tau,\xi,y,t_0\right)\right]d\tau d\xi,\ 0\le x\le t-t_0\le T-x,$$

$$w_{n+1}(t,x,y,t_0)=w_0(t,x,y,t_0)+$$

$$\frac{1}{2}\int\limits_0^x\Big[\triangle v_n\left(t-x+\xi,\xi,y,t_0\right)-\triangle v_n\left(t+x-\xi,\xi,y,t_0\right)$$

$$+a_n\left(t-x+\xi,\xi,y\right)w_n\left(t-x+\xi,\xi,y,t_0\right)-a_n\left(t+x-\xi,\xi,y\right)w_n\left(t+x-\xi,\xi,y,t_0\right)\Big]d\xi d\tau,$$

$$a_{n+1}(t,x,y)=a_0(t,x,y)$$

$$-\frac{4}{g(y)}\int_0^x\left[\triangle v_n\left(t+x-\xi,\xi,y,t-x\right)+a_n(t+x-\xi,\xi,y)w_n(t+x-\xi,\xi,y,t-x)\right]d\xi,$$

$$0\le x\le t\le T-x.$$

Define the function $\tilde{s}_n(x)$ by the formula

$$\tilde{s}_n(x)=\frac{s+\nu^n(x)}{2},\ \nu^n(x)=s_0-\frac{x}{b_n}. \tag{12}$$

Introduce the notations $p_n=v_{n+1}-v_n$, $r_n=w_{n+1}-w_n$, $q_n=a_{n+1}-a_n$, $n=0,\,1,\,2,\ldots$. Then p_n,r_n,q_n satisfy the relations

$$p_0(t,x,y,t_0)=\frac{1}{2}\iint\limits_{\triangle'(t,x)}\left[\triangle v_0\left(\tau,\xi,y,t_0\right)+a_0\left(\tau,\xi,y\right)v_0\left(\tau,\xi,y,t_0\right)\right]d\tau d\xi,$$

$$r_0(t,x,y,t_0)=\frac{1}{2}\int\limits_0^x\Big[\triangle v_0\left(t-x+\xi,\xi,y,t_0\right)-\triangle v_0\left(t+x-\xi,\xi,y,t_0\right)$$

$$+a_0\left(t-x+\xi,\xi,y\right)w_0\left(t-x+\xi,\xi,y,t_0\right)-a_0\left(t+x-\xi,\xi,y\right)w_0\left(t+x-\xi,\xi,y,t_0\right)\Big]d\xi d\tau,$$

$$(t,x,y,t_0)\in D_T,$$

$$q_0(t,x,y)=-\frac{4}{g(y)}\int_0^x\Big\{\triangle v_0\left(t+x-\xi,\xi,y,t-x\right)$$

$$+a_0(t+x-\xi,\xi,y)w_0(t+x-\xi,\xi,y,t-x)\Big\}d\xi,$$

$$p_{n+1}(t,x,y,t_0)=\frac{1}{2}\iint\limits_{\triangle'(t,x)}\Bigg\{\triangle p_n(\tau,\xi,y,t_0)$$

$$+q_n\left(\tau,\xi,y\right)w_{n+1}(\tau,\xi,y,t_0)+a_n\left(\tau,\xi,y\right)r_n\left(\tau,\xi,y,t_0\right)\Bigg\}d\tau d\xi,\quad (t,x,y,t_0)\in D_T,$$

$$r_{n+1}(t,x,y,t_0)=\frac{1}{2}\int\limits_0^x\Big[\triangle p_n\left(t-x+\xi,\xi,y,t_0\right)-\triangle p_n\left(t+x-\xi,\xi,y,t_0\right)$$

$$+q_n\left(t-x+\xi,\xi,y\right)w_{n+1}\left(t-x+\xi,\xi,y,t_0\right)+a_n\left(t-x+\xi,\xi,y\right)r_n\left(t-x+\xi,\xi,y,t_0\right)$$

$$-q_n\left(t+x-\xi,\xi,y\right)w_{n+1}\left(t+x-\xi,\xi,y,t_0\right)-a_n\left(t+x-\xi,\xi,y\right)r_n\left(t+x-\xi,\xi,y,t_0\right)\Big]d\xi d\tau,$$

$$(t,x,y,t_0)\in D_T,$$

$$q_{n+1}(t,x,y)=-\frac{4}{g(y)}\int_0^x\Bigg\{\triangle p_n(t+x-\xi,\xi,y,t-x)$$

$$+q_n(t+x-\xi,\xi,y)w_{n+1}(t+x-\xi,\xi,y,t-x)+a_n(t+x-\xi,\xi,y)r_n(t+x-\xi,\xi,y,t-x)\Bigg\}d\xi,$$

$$(t,x,y)\in G_T\times\mathbb{R}^m.$$

Show that $b_0\in\left(0,\frac{T}{2s_0}\right)$ can be chosen so that the following inequalities be valid for all $n=0,\,1,\,2,\cdots$:

$$\lambda_n=\max\Bigg\{\sup_{(t,x,s)\in\hat{F}_n}\left[\|p_n\|_s(t,x,t_0)\frac{\nu^n(x)-s}{x}\right],\ \sup_{(t,x,s)\in F_n}\left[\|r_n\|_s(t,x,t_0)\frac{(\nu^n(x)-s)^2}{x}\right],$$

$$\sup_{(t,x,s)\in F_n}\left[\|q_n\|_s(t,x)\frac{(\nu^n(x)-s)^2}{x}\right]\Bigg\}<\infty, \tag{13}$$

$$\|v_{n+1}-v_0\|_s(t,x,t_0)\le R,\ \|w_{n+1}-w_0\|_s(t,x)\le\frac{R}{s_0-s},\ \|a_{n+1}-a_0\|_s(t,x)\le\frac{R}{s_0-s}, \tag{14}$$

where

$$\hat{F}_n=\left\{(t,x,t_0,s)|\ (t,x,t_0)\in\Upsilon'_T,\ 0\le x+t_0<b_n(s_0-s),\ 0<s<s_0\right\},$$

$$F_n=\left\{(t,x,s)|\ (t,x)\in G_T,\ 0\le x<b_n(s_0-s),\ 0<s<s_0\right\}.$$

Indeed, using the relations for p_n, q_n, we find

$$\|p_0\|_s(t,x,t_0)\le\frac{1}{2}\iint\limits_{\triangle'(t,x)}\Big[\|\triangle v_0\|_s\left(\tau,\xi,t_0\right)+\|a_0\|_s\left(\tau,\xi\right)\|w_0\|_s\left(\tau,\xi,t_0\right)\Big]d\tau d\xi$$

$$\leq \frac{1}{2} \iint\limits_{\triangle'(t,x)} \left[\frac{Rc_0}{(\tilde{s}_0(\xi) - s)^2} + R^2 \right] d\tau d\xi.$$

Here c_0 is a positive constant, such that

$$\|\triangle v_0\|_s \leq c_0 \frac{\|v_0\|_{\tilde{s}_n}}{(\tilde{s}_n - s)^2}, \ \tilde{s}_n > s > 0, \ n = 0, 1, 2, \ldots.$$

It is easy to check $c_0 = 4m$.

Taking the function $\tilde{s}_n(\xi)$ from (12), for $n = 0$ we have

$$\|p_0\|_s(t, x, t_0) \leq \frac{1}{2} \int_0^x (x - \xi) \left[\frac{4Rc_0}{(\nu^0(\xi) - s)^2} + R^2 \right] d\xi$$

$$\leq \frac{1}{2} R \left[4c_0 + s_0^2 R \right] \int_0^x \frac{(x - \xi) d\xi}{(\nu^0(\xi) - s)^2}$$

$$\leq \frac{1}{2} b_0 R \left[4c_0 + s_0^2 R \right] \frac{x}{\nu^0(x) - s}, \ (t, x, s) \in \hat{F}_0.$$

Proceeding analogously, we obtain

$$\|r_0\|_s(t, x, t_0) \leq \frac{1}{2} \int_0^x \left[\frac{4Rc_0}{(\nu^0(\xi) - s)^2} + R^2 \right] d\xi \leq$$

$$\leq \frac{1}{2} R \left(4c_0 + s_0^2 R \right) \frac{x}{(\nu^0(x) - s)^2}, \ (t, x, s) \in \hat{F}_0,$$

$$\|q_0\|_s(t, x) \leq 4g_0 \int_0^x \left[\frac{4Rc_0}{(\nu^0(\xi) - s)^2} + R^2 \right] d\xi \leq$$

$$\leq 4g_0 R \left(4c_0 + s_0^2 R \right) \frac{x}{(\nu^0(x) - s)^2}, \ (t, x, s) \in F_0.$$

These estimates imply validity of inequality (12) for $n = 0$. Moreover,we find

$$\|v_1 - v_0\|_s(t, x, t_0) = \|p_0\|_s(t, x, t_0) \leq \frac{\lambda_0 x}{\nu^0(x) - s} \leq \frac{\lambda_0 b_1}{1 - b_1/b_0} = b_0 \lambda_0, \ (t, x, t_0, s) \in \hat{F}_1,$$

$$\|w_1 - w_0\|_s(t, x, t_0) = \|w_0\|_s(t, x) \leq \frac{\lambda_0 x}{(\nu^0(x) - s)^2} \leq \frac{4b_0 \lambda_0}{s_0 - s}, \ (t, x, s) \in \hat{F}_1,$$

$$\|a_1 - a_0\|_s(t, x) = \|a_0\|_s(t, x) \leq \frac{\lambda_0 x}{(\nu^0(x) - s)^2} \leq \frac{4b_0 \lambda_0}{s_0 - s}, \ (t, x, s) \in F_1.$$

Choosing b_0 so as to have $4b_0 \lambda_0 \leq R$, we conclude that inequalities (14) are satisfied for $n = 0$.

By way of induction, we show that inequalities (13) and (14) are also valid for the other values of n if b_0 is chosen suitably. Assume that inequalities (13) and (14) hold for $n = 0, 1, 2, \ldots, i$. Then, $(t, x, t_0, s) \in \hat{F}_{i+1}$ we have

$$\|p_{i+1}\|_s(t,x,t_0) \le \frac{1}{2} \iint\limits_{\triangle'(t,x)} \Bigg\{ \left\|\triangle p_i\right\|_s(\tau,\xi,t_0)$$

$$+\left\|q_i\right\|_s(\tau,\xi)\left\|w_{i+1}\right\|_s(\tau,\xi,t_0) + \left\|a_i\right\|_s(\tau,\xi)\left\|r_i\right\|_s(\tau,\xi,t_0)\Bigg\} d\tau d\xi$$

$$\le \frac{1}{2} \iint\limits_{\Delta(t,x)} \left[\frac{c_0\lambda_i\xi}{(s_i'(\xi)-s)^2(\nu^i(\xi)-s)} + \frac{\lambda_i\xi}{(\nu^i(\xi)-s)^2}\frac{R(1+s_0)}{(s_0-s)}\right.$$

$$\left.+\frac{\lambda_i\xi}{(\nu^i(\xi)-s)^2}\frac{R(1+s_0)}{(s_0-s)}\right] d\tau d\xi \le \frac{\lambda_i}{2}\left(4c_0+2R(1+s_0)\right)\int_0^x \frac{(x-\xi)\xi d\xi}{(\nu^{i+1}(\xi)-s)^3}$$

$$\le \frac{\lambda_i}{2} b_0^2 \left(4c_0+2R(1+s_0)\right)\frac{x}{\nu^{i+1}(x)-s}.$$

Here in the intermediate calculations, the function s_i' is defined by equality (11) with $n=i$ and the inequalities

$$\|w_i\|_s(t,x,t_0) \le R\frac{1+s_0}{s_0-s},\ \ \|a_i\|_s(t,x) \le R\frac{1+s_0}{s_0-s}$$

are used, the latter valid by the induction hypothesis, together with the obvious inequalities $b_i \le b_0$ and $\nu^{i+1}(x) \le \nu^i(x)$. Similar arguments for r_{i+1}, q_{i+1} lead to inequalities

$$\|r_{i+1}\|_s(t,x,t_0) \le \frac{1}{2}\int_0^x \Bigg\{ \frac{2c_0\lambda_i\xi}{(\tilde{s}_i(\xi)-s)^2(\nu^i(\xi)-s)} + \frac{2\lambda_i\xi}{(\nu^i(\xi)-s)^2}\frac{R(1+s_0)}{(s_0-s)}$$

$$+\frac{2\lambda_i\xi}{(\nu^i(\xi)-s)^2}\frac{R(1+s_0)}{(s_0-s)}\Bigg\} d\xi \le \lambda_i\left[4c_0+2R(1+s_0)\right]\int_0^x \frac{\xi d\xi}{(\nu^{i+1}(\xi)-s)^3}$$

$$\le \lambda_i b_0\left[4c_0+2R(1+s_0)\right]\frac{x}{(\nu^{i+1}(x)-s)^2},\ \ (t,x,s)\in \hat{F}_{i+1},$$

$$\|q_{i+1}\|_s(t,x) \le 4g_0\int_0^x \Bigg\{ \frac{c_0\lambda_i\xi}{(\tilde{s}_i(\xi)-s)^2(\nu^i(\xi)-s)} + \frac{\lambda_i\xi}{(\nu^i(\xi)-s)^2}\frac{R(1+s_0)}{(s_0-s)}$$

$$+\frac{\lambda_i\xi}{(\nu^i(\xi)-s)^2}\frac{R(1+s_0)}{(s_0-s)}\Bigg\} d\xi \le 4\lambda_i g_0\left[4c_0+2R(1+s_0)\right]\int_0^x \frac{\xi d\xi}{(\nu^{i+1}(\xi)-s)^3}$$

$$\le 4\lambda_i g_0 b_0\left[4c_0+2R(1+s_0)\right]\frac{x}{(\nu^{i+1}(x)-s)^2},\ \ (t,x,s)\in F_{i+1}.$$

The obtained estimates yield

$$\lambda_{i+1} \le \lambda_i\rho,\ \ \lambda_{i+1} < \infty,$$

$$\rho = b_0\left[4c_0+2R(1+s_0)\right]\max\left[\frac{b_0}{2}, 4g_0, 1\right].$$

Moreover, we have

$$\|v_{i+2}-v_0\|_s(t,x,t_0) \le \sum_{n=0}^{i+1} \|p_n\|_s(t,x,t_0) \le \sum_{n=0}^{i+1} \frac{\lambda_n x}{\nu^n(x)-s} \le \sum_{n=0}^{i+1} \frac{\lambda_n b_{i+2}}{1-b_{i+2}/b_n}$$

$$\le \sum_{n=0}^{i+1} \lambda_n b_n (n+1)^2 \le \lambda_0 b_0 \sum_{n=0}^{i+1} \rho^n (n+1)^2,\ (t,x,t_0,s) \in \hat{F}_{i+2},$$

$$\|w_{i+2}-w_0\|_s(t,x,t_0) \le \sum_{n=0}^{i+1} \|r_n\|_s(t,x,t_0) \le \sum_{n=0}^{i+1} \frac{\lambda_n x}{(\nu^n(x)-s)^2} \le \frac{1}{s_0-s} \sum_{n=0}^{i+1} \frac{\lambda_n b_{i+2}}{(1-b_{i+2}/b_n)^2} \le$$

$$\le \frac{\lambda_0 b_0}{s_0-s} \sum_{n=0}^{i+1} \rho^n (n+1)^4,\ (t,x,t_0,s) \in \hat{F}_{i+2},$$

$$\|a_{i+2}-a_0\|_s(t,x) \le \sum_{n=0}^{i+1} \|q_n\|_s(t,x) \le \sum_{n=0}^{i+1} \frac{\lambda_n x}{(\nu^n(x)-s)^2} \le \frac{1}{s_0-s} \sum_{n=0}^{i+1} \frac{\lambda_n b_{i+2}}{(1-b_{i+2}/b_n)^2} \le$$

$$\le \frac{\lambda_0 b_0}{s_0-s} \sum_{n=0}^{i+1} \rho^n (n+1)^4,\ (t,x,s) \in F_{i+2}.$$

Now we choose $b_0 \in \left(0, \frac{T}{2s_0}\right)$ so as to obtain

$$\rho < 1,\ \lambda_0 b_0 \sum_{n=0}^{\infty} \rho^n (n+1)^4 \le R.$$

Then

$$\|v_{i+2}-v_0\|_s(t,x,t_0) \le R,\ \|w_{i+2}-w_0\|_s(t,x,t_0) \le \frac{R}{s_0-s}, \quad (t,x,t_0,s) \in \hat{F}_{i+2},$$

$$\|a_{i+2}-a_0\|_s(t,x) \le \frac{R}{s_0-s},\ (t,x,s) \in F_{i+2}.$$

Since the choice of b_0 is independent of the number of the approximation, all the successive approximations v_n, w_n, a_n belong to

$$C_{(t,x,t_0)}\left(\hat{F};\ A_s\right),\ \hat{F} = \bigcap_{n=0}^{\infty} \hat{F}_n,$$

and

$$C_{(t,x)}\left(F;\ A_s\right),\ F = \bigcap_{n=0}^{\infty} F_n,$$

respectively. Moreover,

$$\|v_n-v_0\|_s(t,x,t_0) \le R,\ \|w_n-w_0\|_s(t,x,t_0) \le \frac{R}{s_0-s}, (t,x,t_0,s) \in \hat{F},$$

$$\|a_n - a_0\|_s(t,x) \le \frac{R}{s_0 - s}, \ (t,x,s) \in F.$$

For $s \in (0, s_0)$ the series

$$\sum_{n=0}^{\infty}(v_n - v_{n-1}), \ \sum_{n=0}^{\infty}(w_n - w_{n-1}), \ \sum_{n=0}^{\infty}(a_n - a_{n-1})$$

converge uniformly in the norm of the spaces

$$C_{(t,x,t_0)}\left(P_{sT};\ A_s\right),\ P_{sT} = \Upsilon'_T \cap \left\{(t,x,t_0) : 0 \le x + t_0 < b(s_0 - s)\right\},$$

$$C_{(t,x)}\left(K_{sT};\ A_s\right),\ K_{sT} = G_T \cap \left\{(t,x,t_0) : 0 \le x + t_0 < b(s_0 - s)\right\},$$

therefore $v_n \to v$, $w_n \to w$, $a_n \to a$, and the limit functions v, w, a are elements of $C_{(t,x,t_0)}\left(P_{sT};\ A_s\right)$, $C_{(t,x)}\left(K_{sT};\ A_s\right)$ respectively and satisfy equations (9)–(11).

Now we prove that this solution is unique. Let (v, w, a) and $(\widehat{v}, \widehat{w}, \widehat{a})$ be any two solutions satisfying the inequalities

$$\|v - v_0\|_s(t,x,t_0) \le R, \ \|w - w_0\|_s(t,x,t_0) \le \frac{R}{s_0 - s}, \ (t,x,t_0,s) \in \hat{F},$$

$$\|a - a_0\|_s(t,x) \le \frac{R}{s_0 - s}, \ (t,x,s) \in F.$$

Denote $\tilde{p} = v - \widehat{v}$, $\tilde{r} = r - \widehat{r}$, $\tilde{q} = a - \widehat{a}$,

$$\lambda := \max\left\{\sup_{(t,x,t_0,s)\in\hat{F}}\left[\|\tilde{p}\|_s(t,x,t_0)\frac{\nu(x) - s}{x}\right], \sup_{(t,x,t_0,s)\in\hat{F}}\left[\|\tilde{r}\|_s(t,x,t_0)\frac{(\nu(x) - s)^2}{x}\right],\right.$$

$$\left.\sup_{(t,x,s)\in F}\left[\|\tilde{q}\|_s(t,x)\frac{(\nu(x) - s)^2}{x}\right]\right\} < \infty,$$

where $\nu(x) = s_0 - x/b$, $b = b_0 \prod\limits_{n=0}^{\infty}\left(1 + 1/(n+1)^2\right)^{-1}$. Then, for the functions $\tilde{p}$, $\tilde{q}$ can be obtained the relations

$$\tilde{p}(t,x,y,t_0) = \frac{1}{2}\iint\limits_{\triangle'(t,x)}\left\{\triangle\tilde{p}\left(\tau,\xi,y,t_0\right)\right.$$

$$\left.+\tilde{q}\left(\tau,\xi,y\right)\widehat{w}(\tau,\xi,y,t_0) + a\left(\tau,\xi,y\right)\tilde{r}\left(\tau,\xi,y,t_0\right)\right\}d\tau d\xi, \ (t,x,y,t_0) \in D_T,$$

$$\tilde{r}(t,x,y,t_0) = \frac{1}{2}\int_0^x\left[\triangle\tilde{p}\left(t - x + \xi, \xi, y, t_0\right) - \triangle\tilde{p}\left(t + x - \xi, \xi, y, t_0\right)\right.$$

$$+\tilde{q}\left(t - x + \xi, \xi, y\right)\widehat{w}\left(t - x + \xi, \xi, y, t_0\right) + a\left(t - x + \xi, \xi, y\right)\tilde{r}\left(t - x + \xi, \xi, y, t_0\right)$$

$$-\tilde{q}\,(t+x-\xi,\xi,y)\,\widehat{w}\,(t+x-\xi,\xi,y,t_0)-a\,(t+x-\xi,\xi,y)\,\tilde{r}\,(t+x-\xi,\xi,y,t_0)\Big]\,d\xi d\tau,$$

$$(t,x,y,t_0)\in D_T,$$

$$\tilde{q}(t,x,y)=-\frac{4}{g(y)}\int_0^x\Big\{\triangle\tilde{p}\,(t+x-\xi,\xi,y,t-x)$$

$$+\tilde{q}(t+x-\xi,\xi,y)\widehat{w}(t+x-\xi,\xi,y,t-x)+a(t+x-\xi,\xi,y)\tilde{r}(t+x-\xi,\xi,y,t-x)\Big\}d\xi,$$

$$(t,x,y)\in G_T\times\mathbb{R}^m.$$

Show that $b_0\in\left(0,\frac{T}{2s_0}\right)$ can be chosen so that the following inequalities be valid for all $n=0,\,1,\,2,\ldots.$ Applying to these equations the estimates given above, we find the inequality in the form

$$\lambda\le\lambda\rho',$$

$$\rho':=b\,[4c_0+2R(1+s_0)]\max\left[\frac{b}{2},4g_0,1\right]<\rho<1.$$

Consequently $\lambda=0$. Therefore $v=\widehat{v}$, $w=\widehat{w}$,$a=\widehat{a}$. Theorem 1 is proved.

Consider the set Γ of functions $f(t,y,t_0)$ representing the elements of $C(Q_T;\,A_{s_0}),\ s_0>0$, for which the conditions of Theorem 1 are valid with R, T, s_0. Then we have the stability theorem

Theorem 2. *Let $f,\ \bar{f}\in\Gamma$. For the corresponding solutions (v,w,a) and $(\bar{v},\bar{w},\bar{a})$ to (9)–(11), we have*

$$\begin{gathered}\|v-\bar{v}\|_s\le cM,\ \|w-\bar{w}\|_s\le\frac{cM}{s_0-s},\ (t,x,t_0)\in P_{sT},\\ \|a-\bar{a}\|_s\le\frac{cM}{s_0-s},\ (t,x)\in K_{sT},\ 0<s<s_0,\end{gathered}\tag{15}$$

where

$$M=\max\big[\max\|f-\bar{f}\|_{s_0}(t,t_0),\ \max\|f_t-\bar{f}_t\|_{s_0}(t,t_0)\big],\ (t,t_0)\in Q_T,$$

and the constant c depends on R, T, s_0.

Proof. For the differences $v-\bar{v}=\tilde{v},\ a-\bar{a}=\tilde{a}$ and $f-\bar{f}=\tilde{f}$ the equalities follow from (9)–(10):

$$\tilde{v}(t,x,y,t_0)=\tilde{v}_0(t,x,y,t_0)+\frac{1}{2}\iint\limits_{\triangle'(t,x)}\Big\{\triangle\tilde{v}\,(\tau,\xi,y,t_0)$$

$$+\tilde{a}\,(\tau,\xi,y)\,w(\tau,\xi,y,t_0)+\bar{a}\,(\tau,\xi,y)\,\tilde{w}\,(\tau,\xi,y,t_0)\Big\}\,d\tau d\xi,\ (t,x,y,t_0)\in D_T,\tag{16}$$

$$\tilde{w}(t,x,y,t_0) = \tilde{w}_0(t,x,y,t_0) + \frac{1}{2}\int\limits_0^x \Big[\triangle\tilde{v}\left(t-x+\xi,\xi,y,t_0\right) - \triangle\tilde{v}\left(t+x-\xi,\xi,y,t_0\right)$$

$$+\tilde{a}\left(t-x+\xi,\xi,y\right)w\left(t-x+\xi,\xi,y,t_0\right) + \bar{a}\left(t-x+\xi,\xi,y\right)\tilde{w}\left(t-x+\xi,\xi,y,t_0\right)$$

$$-\tilde{a}\left(t+x-\xi,\xi,y\right)w\left(t+x-\xi,\xi,y,t_0\right) - \bar{a}\left(t+x-\xi,\xi,y\right)\tilde{w}\left(t+x-\xi,\xi,y,t_0\right)\Big]d\xi d\tau, \tag{17}$$

$$\tilde{a}(t,x,y) = \tilde{a}_0(t,x,y) - \frac{4}{g(y)}\int_0^x \Big\{\triangle\tilde{v}\left(t+x-\xi,\xi,y,t-x\right)$$

$$+\tilde{a}(t+x-\xi,\xi,y)w(t+x-\xi,\xi,y,t-x)+\bar{a}(t+x-\xi,\xi,y)\tilde{w}(t+x-\xi,\xi,y,t-x)\Big\}d\xi, \tag{18}$$

$$(t,x,y) \in G_T \times \mathbb{R}^m,$$

where

$$\tilde{v}_0(t,x,y,t_0) = \frac{1}{2}[\tilde{f}(t+x,y,t_0) + \tilde{f}(t-x,y,t_0)],$$

$$\tilde{w}_0(t,x,y,t_0) = \frac{1}{2}[\tilde{f}'_t(t+x,y,t_0) + \tilde{f}'_t(t-x,y,t_0)],$$

$$\tilde{a}_0(t,x,y) = \frac{4}{g(y)}\tilde{f}_t(t+x,y,t-x).$$

It is obvious that

$$\|\tilde{v}_0\|_{s_0}(t,x,t_0) \le M,\ \|\tilde{w}_0\|_{s_0}(t,x,t_0) \le M,\ (t,x,t_0) \in P_{sT},$$

$$\|\tilde{a}_0\|_{s_0}(t,x) \le \frac{4}{\|g(y)\|_{s_0}}M,\ (t,x) \in K_{sT}, \tag{19}$$

From Theorem 1 the estimates follow:

$$\|v\|_s \le 2R,\ \|w\|_s \le \frac{R(1+s_0)}{s_0-s},\ \|a\|_s \le \frac{R(1+s_0)}{s_0-s}.$$

Applying the method of successive approximations used for the proof of Theorem 1 to the system of equations (16)–(18), which is linear with respect to $\tilde{v}$ and $\tilde{a}$, we find that, the following inequalities are valid for solution to (16)–(18):

$$\|\tilde{v}-\tilde{v}_0\|_s(t,x,t_0) \le c_1M,\ \|\tilde{w}-\tilde{w}_0\|_s(t,x,t_0) \le \frac{c_1M}{s_0-s}, (t,x,t_0) \in P_{sT},$$

$$\|\tilde{a}-\tilde{a}_0\|_s(t,x) \le \frac{c_1M}{s_0-s},\ (t,x) \in K_{sT},\ 0 < s < s_0.$$

where c_1 depends on R, T, s_0. Hence, by (19) inequalities (15) follow. Theorem 2 is proven.

4 References

1. Romanov V. G. *Stability in Inverse Problems* [Russian], Nauchnyi Mir, Moscow (2005).

2. Ovsyannikov L. V. A singular operator in a scale of Banach spaces, *Soviet Math. Dokl.*, vol. 6, no. 4, 1025–1028 (1965).

3. Ovsyannikov L. V. A nonlocal Cauchy problem in scales of Banach spaces, *Dokl. Akad. Nauk SSSR*, vol. 200, no. 4, 789–792 (1971).

4. Nirenberg L. *Topics in Nonlinear Functional Analysis*, New York: Courant Institute Math. Sci., New York Univ., 1974.

5. Romanov V. G. Local solvability of some multidimensional inverse problems for hyperbolic equations, *Differ. Equ.*, vol. 25, no. 2, 203–209 (1989). http://www.mathnet.ru/links/7b1533bb6958f99f23cf0c9fcfb3ef57/de6759.pdf

6. Romanov V. G. On solvability of inverse problems for hyperbolic equations in a class of functions analytic in part of variables, *Dokl. Akad. Nauk SSSR*, vol. 304, no. 4, 807–811 (1989). https://zbmath.org/0682.35105

7. Durdiev D. K. and Nuriddinov Z. Z. Determination of a multidimensional kernel in some parabolic integro–differential equation, *Journal of Siberian Federal University-Mathematics and Physics*, 14:1 (2021), 117–127. https://doi.org/10.17516/1997-1397-2021-14-1-117-127

8. Bozorov Z. R., The problem of determining the two-dimensional kernel of a viscoelasticity equation, *Sib. Zh. Ind. Mat.*, vol. 23, no. 1, 28–45 (2020). https://doi.org/10.33048/SIBJIM.2020.23.104

9. Durdiev D. K. and Rahmonov A. A. The problem of determining the 2D kernel in a system of integro-differential equations of a viscoelastic porous medium, *J. Appl. Indust. Math.*, vol. 14, no. 2, 281–295 (2020). https://doi.org/10.33048/SIBJIM.2020.23.205

10. Durdiev D. K. and Totieva Zh. D. The problem of determining the multidimensional kernel of the viscoelasticity equation, *Vladikavkaz. Mat. Zh.*, vol. 17, no. 4, 18–43 (2015). https://zbmath.org/?q=an:1474.45100

11. Durdiev D. K. and Safarov Zh. Sh. Local solvability of the problem of definition of the spatial part of the multidimensional kernel in an integro-differential equation of hyperbolic type, *Vestnik Samarsk. Gos. Univ. Ser. Fiz.-Mat. Nauki*, vol. 29, no. 4, 37–47 (2012).

12. Durdiev D. K. Some multidimensional inverse problems of memory determination in hyperbolic equations, Zh. Mat. Fiz., Anal. Geom., vol. 3, no. 4, 411–423 (2007). https://zbmath.org/?q=an:06137633

13. Durdiev D. K. A multidimensional inverse problem for an equation with memory, *Sib. Math. J.*, vol. 35, no. 3, 514–521 (1994).