REPORT

on the paper

Stanislav O. Speranski,
Notes on Došen's logic N *and its extensions.*

The paper combines some results on extensions of Došen's logic N [2]. The idea behind N is to consider a modal operator of impossibility over the positive fragment of intuitionistic logic. This results in a negation (hence the title of [2]) which is weaker than that of intuitionistic logic IL or even that of minimal Johansson's logic JL. In his paper Došen introduces Kripke style semantics for his systems and develops a bit of correspondence theory to show, how exactly well known semantics of IL and JL can be obtained by extending N.

Aside from N itself, the author considers its three known extensions: N° [6], N* [1, 5] and N•
[3, 4]. Note that these four logics are related as follows:

$$\mathsf{N} \quad \subseteq \quad \mathsf{N}° \quad \subseteq \quad \mathsf{N}^* \quad \subseteq \quad \mathsf{N}^•.$$

This paper essentially consists of three independent parts. The first one (Section 3) is dedicated to extending three extensions of N above with contraposition in the axiom form (all of them satisfy contraposition in the rule form). The second one (Section 4) covers some questions regarding the correspondence theory and canonicity for formulas, comprising the axiomatization of N•. Finally, the third one (Section 5) develops filtration method that can be adapted for logics under consideration.

The results overall are fairly simple yet provide some interesting points on the considered logics. I do find that some of the results are fairly selective in their scope and could be expanded considerably. I make few suggestions on that in the remarks below. I do not feel like it would be fair to force the author to make substantive changes given that there are no particular problems with the contents presented but I also want to leave a room for the author to implement these changes if he sees fit. Thus I recommend to conduct a second round of reviews after the revision.

My biggest suggestion has to do with the choice of semantics for systems presented in the paper. The author chooses to use the semantics as presented in [2], which might seem like a natural choice but is also—I think—a quite limiting one. There is an alternative semantics using sets of normal worlds given, e.g. in [7]. This semantics involve one additional element: an up-set of *normal worlds* N and has exactly the same validity conditions as in [2], except for (in author's notation):

$$\mathcal{M}, x \Vdash \neg\psi \iff (\mathcal{M}, y \nVdash \psi \text{ for all } y \in R(x)) \text{ and } x \in N.$$

Looking through the results in [7] it looks like $N1°$ corresponds to $\forall x(x \in N \implies \exists y\, R(x,y))$ with respect to this semantics. And unless I'm wrong, $N2°$ corresponds to

$$\forall x\ (x \in N) \text{ and } \forall x(\exists y\, R(y,x) \implies \exists z\, R(x,z)).$$

This, in particular, means that $N2°$ is no longer "semantically weaker than $N1°$" (a claim on P. 9) with respect to this semantics, which, in turn, blocks the proof of Proposition 4.1. Overall, I would expect that all of the formulas considered in the paper (including possibly E?) would be canonical schemes with respect to the semantics above, which would paint a much broader picture. Naturally, committing to semantics with non-normal worlds would have a lot of repercussions for

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the contents of the paper (for instance, one would have to figure out whether the filtration method could be adopted for this semantics), which is why I do not want to force the author to do so, but at the very least the possibility of considering this semantics should be given in paper along with some of it consequences.

Below are more specific comments.

1. P. 4. I think it is worth pointing out to the reader that $\mathsf{N}^\bullet$ as presented here is a definitional variant of the system from [3] (which has the truth constant in the language).

2. P. 5. I feel like the statement of Proposition 2.3 should be strengthen to the effect that for any extension L of N logics $L + \{C\}$ and $L + \{E\}$ coincide. Note that this follows immediately from the proof provided in the paper.

3. P. 8. "Strongly condensed" $\mapsto$ "strictly condensed". Similarly in the proof of Proposition 5.1. I also think it is worth pointing out that this is an equivalent definition of strictly condensed frames from [2].

4. P. 9, "From now on we shall assume that our prime theories are non-trivial". The frequent switching between allowing and disallowing trivial theories seems very confusing to me, especially since the same notation is used for both. I would suggest to introduce a different notation for $W^L \setminus \{\mathsf{Form}\}$ (or just use that) and add the designation "non-trivial" explicitly to every statement that is subject to the assumption that only non-trivial prime theories are considered.

5. P. 9, Section 3. I think the author should give some explanation as to why only logic containing $N2^\circ$ are considered in this section. In particular, why the logic $\mathsf{N} + \{E\}$ is not investigated at all.

6. P. 10, Lemma 3.1. It would be nice to point out that according to [2], JL can be presented as $\mathsf{N} + \{E, N1^\bullet\}$. This will give the reader a better understanding of the result in Lemma 3.1.

7. P. 11, Section 4. For the purpose of this section it is worth mentioning that the presence of $N2^\circ$ guaranties that all $\underline{\Gamma}$ are non-empty, which is sometimes needed for the applications of Lemma 2.6.

8. P. 12, "treated as variable-free". It would help to comment that this means that the validity of formula does not depend on the choice of a valuation.

9. P. 13, Corollary 4.3. Probably, $\cup \mapsto +$.

10. P. 14, "finest and coarsest Φ-filtrations". I wonder whether these filtrations deserve their names. Is it the case that

$$\leq_\Phi \subseteq \leq' \subseteq \leq^\Phi \qquad \text{and} \qquad R_\Phi \subseteq R \subseteq R^\Phi$$

for any filtration $\langle W_\Phi, \leq', R' \rangle$?

11. P. 15. Is $\mathcal{W}^\Phi$ strictly condensed? Or does it at least preserve the property of being strictly condensed?

12. P. 16, "$N(\Phi)$ may be treated as finite modulo $\mathsf{N} + \{N1^\bullet\}$". I feel like it would be easier to say that $\mathcal{W}^{N(\Phi)}$ would be finite even though $N(\Phi)$ is infinite, due to the considerations given (in case Φ itself is finite).

13. P. 17, "Moreover, it can be treated as a finite set". In this case I would love to see a more careful explanation on how we arrive to a finite set of formulas. At the very least, note that all of the comments on the finite nature of $C(\Phi)$ are only given after the proof of Lemma 5.10, which employs these considerations.

References

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