Referee's report on "The vertex connectivity of some classes of divisible design graphs"

In the paper under review, the author studied the vertex connectivity of divisible design graphs. Divisible design graphs have been studied and constructed by Meulenberg, Haemers, and Kharaghani, and is an example of Deza graphs. The vertex connectivity for Deza graphs behaves differently as strongly regular graphs. The author determines the vertex connectivity for constructed DDG by aforementioned authors. The most important result in this paper is to determine the vertex connectivity for DDG from regular graphical Hadamard matrices, which shows that the resulting DDG has the vertex conenctivity much smaller than its degree. I would like to recommend this paper to be accepted with minor revision.

The below are some comments.

(1) p148, Lemma 8 (3), "G is 3×3-lattice" should be "G is a 3×3-lattice".

(2) p154, Lemma 16, The proof of this lemma is lengthy. An alternative proof is the following:

The adjacency matrix of Γ_1^t is M, where the notation is from Construction 6. Then, by $HJ = JH = lJ$ and $H^2 = l^2 I$,

$$\begin{aligned} M^2 &= \frac{1}{4}\begin{bmatrix} J+H & J+H \\ J+H & J+H \end{bmatrix}^2 \\ &= \frac{1}{4}\begin{bmatrix} 2(J+H)^2 & 2(J+H)^2 \\ 2(J+H)^2 & 2(J+H)^2 \end{bmatrix} \\ &= \frac{1}{4}\begin{bmatrix} 2(J^2+HJ+JH+H^2) & 2(J^2+HJ+JH+H^2) \\ 2(J^2+HJ+JH+H^2) & 2(J^2+HJ+JH+H^2) \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} (l^2+2l)J+l^2 I & (l^2+2l)J+l^2 I \\ (l^2+2l)J+l^2 I & (l^2+2l)J+l^2 I \end{bmatrix}, \end{aligned}$$

which is permutation-equivalent to $\frac{l^2}{2} I_{l^2} \otimes J_2 + \frac{l^2+2l}{2} J_{2\ell^2}$. Thus, Γ_1^t is a DDG with parameters $(2\ell^2, l^2+l, l^2+l, (l^2+2l)/2, l^2, 2)$.

(3) p155, Lemma 17, Similar to the above comment, we may have a shorter proof like this:

The adjacency matrix of Γ_2^t is N. Then, by $HJ = JH = lJ$ and

$$H^2 = l^2 I,$$

$$\begin{aligned} N^2 &= \frac{1}{4}\begin{bmatrix} J+H & J-H \\ J-H & J+H \end{bmatrix}^2 \\ &= \frac{1}{4}\begin{bmatrix} (J+H)^2+(J-H)^2 & (J+H)(J-H)+(J-H)(J+H) \\ (J+H)(J-H)+(J-H)(J+H) & (J+H)^2+(J-H)^2 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} l^2(J+I) & l^2(J-I) \\ l^2(J-I) & l^2(J+I) \end{bmatrix}, \end{aligned}$$

which is permutation-equivalent to $\frac{l^2}{2} I_{2l^2} - \frac{l^2}{2} I_{2l^2} \otimes (J_2 - I_2) + \frac{l^2}{2} J_{2l^2}$. Thus, Γ_2^t is a DDG with parameters $(2l^2, l^2, 0, l^2/2, l^2, 2)$.

(4) If the above comments are agreeable, then Lemma 13, Lemma 14, and Lemma 15 seem not needed.

(5) p157, Proof ofLemma 18, line-1,2, please delete "$\sum_{x \in B} \sum_{y \in B \setminus \{x\}} (|S(x)| + |S(y)|)$" in the left hand side.

(6) p158, Lemma 20 should be Theorem 20.