REPORT
on the paper
"Description of modal logics which enjoy co-cover property"
by V.V. Rimatskiy.

This short paper is dedicated to providing a syntactic counterpart of so called weak co-cover property in terms of admissibility of rules. More specifically, the main result of the paper states that an extension of normal modal logic S4 satisfying the finite model property has weak co-cover property iff all inference rules from a certain countable set are admissible in it.

Unfortunately a bit of work still has to be done before this paper's publication. I am attaching the pdf-file of the paper annotated with remarks. Among these yellow highlights are language-based suggestions and corrections (as I am not a native speaker of English, I recommend taking these with a grain of salt); orange are technical remarks. The main technical remarks are also listed below. It seems to me that non of these will be difficult to resolve.

1. In the definition of $Ch_n(\lambda)$ one restriction from [1] is missing. Namely that no two elements in the same cluster should be evaluated the same way.

2. In the proof of theorem 1 there is still no mention of the case, when $\{C(b_1), \ldots, C(b_n)\}$ forms a trivial antichain of clusters, which would render WCP not applicable.

3. I am afraid my previous suggestion to the proof of proposition 3 was slightly misguiding. From the fact that $C(z)$ is not a co-cover of $\mathcal{X}$ the existence of y with $y \notin \mathcal{X}^R \cup \mathcal{X}^{-R}$ and zRy does not follow (z might be just below a co-cover). It does feel like the definition of the valuation should work, since there has to be some $y \geq b$ such that $\mathcal{X} \cup C(y)$ forms an antichain (otherwise the result of adding a singleton co-cover to $\mathcal{X}$ would be a p-morphic image of $\mathcal{M}$), in which case $y \in v(q)$, $b \in \mathcal{X}^{-R}$ and xRy, but I urge the author to properly check this proof (or come up with another) and fill in all the formal details.

4. The claim in proposition 4 that $\mathcal{M}_0$ is an open submodel of some $Ch_k(\lambda)$ is wrong. Since $\mathcal{M}_0$ was an arbitrary rooted finite frame it might have certain duplicates (of clusters and elements within a cluster) that all $Ch_k(\lambda)$ specifically avoid. In fact, the author refers to the proof of theorem 3.3.6 in [1], which does not claim this at all. What it does instead is show that there is an open submodel of $Ch_k(\lambda)$ which is a p-morphic image of $\mathcal{M}_0$. These details have to clarified.

5. The proof of proposition 4 also contains the following claim: "From this by construction of λ-successor $\mathcal{M}$ we infer that the antichain of minimal clusters from $f(\mathcal{A}^R)$ has singleton reflexive co-cover ϵ in $\mathcal{M}$". This only holds if $f(\mathcal{A})$ forms a **non-trivial** antichain. What if it forms a trivial one?

6. Finally, the end of the proof of proposition 4 should contain an explanation (or a reference) as to why $f^{-1}(V)$ is a definable valuation on $Ch_k(\lambda)$, which is what we need to apply theorem 4.

References

[1] V. V. Rybakov. *Admissibility of Logical Inference Rules.* ELSEVIER LTD, 1997.

[2] V. V. Rybakov. Construction of an explicit basis for rules admissible in modal system s4. *MLQ*, 47(4):441–446, nov 2001.