

Review on the paper

Nadia Mohdeb “Stability analysis of a prey-predator population model with nonlinear harvesting rate”

In the paper, a prey-predator fishery model with nonlinear harvesting rate is considered. The model is a modification of the well-known Lotka-Volterra prey-predator model. The author investigates the dynamical behavior of the model. The questions concerning the existence and global stability of multiply equilibria are discussed. Some characteristic properties of the equilibria (such that strong and center manifolds) are analyzed. The existence of a limit cycle is studied. Results of computer modeling illustrated constructions under discussion are presented. In particular, bifurcations diagrams are shown. The paper is devoted to the actual problems. Along with this fact, it is necessary to make some essential remarks.

1. The novelty of the work is unclear. In the paper

T. Das, R.N. Mukherje, and K.S. Chaudhuri, Harvesting of a prey–predator fishery in the presence of toxicity // Applied Mathematical Modelling. 2009. Vol. 33, No. 5. P. 2282-2292,

the authors consider system (2), which can be transformed to the GLV model of the form

$$\begin{aligned}\dot{x}_1 &= x_1(a_1 + a_2x_1 + a_3x_2 + a_4x_1^2), \\ \dot{x}_2 &= x_2(b_1 + b_2x_1 + b_3x_2),\end{aligned}$$

while the system under consideration in the paper in question can be rewritten as

$$\begin{aligned}\dot{x} &= x(a_1 + a_2x + a_3y), \\ \dot{y} &= y(b_1 + b_2x + b_3y^2),\end{aligned}$$

which is simpler than previous. The author uses the same technique of proofs to obtain similar steady points and their stability, but does not compare the results. Moreover, all these cases are special forms of the GLV model of the paper

A. Ghasemabadi, Stability and bifurcation in a generalized delay prey–predator model // Nonlinear Dynamics, 2017. Vol. 90. P. 2239-2251,

which also is not discussed.

2. The statement of the problem does not match with the content of the paper. For example, the positive invariant set D is considered on page 3, but some equilibrium points have negative coordinates in Theorem 3.1, which contradicts to the biological interpretation of the LV model, and then the author analyses their stability. Thus, the motivation of the research conducted in the paper is erroneously named as the study of a dynamical behavior of the LV model.

3. On p. 5, Δ_3 appears but it is discussed on p. 7.

4. References [7] and [8,16] (see p.5) should be more precise since they are related to rather large monographs.

5. It is necessary to define the notions “analytically conjugate”, “ C^∞ -conjugate”, and “ C^0 -conjugate”.

6. On p. 5, there are values \bar{x}_0, \bar{y}_0 . What are they?

7. The functions I_1 on p. 6 and I_2 on p. 7 do not depend on x .

8. On p. 6 (row 11 from below), there is the phrase “We can write Δ_2 which is positive”. Why so?

9. It should be clarified how the expression for $\det J(x, y)$ is obtained (see p. 7). Here, the statement “ Δ_3 is positive” has no any substantiation.

10. Multiplier 2 is missing in the right-hand part of (11).

11. The text given in rows 14,15 on the bottom of p.7 requires clarification. For example, why is “the term in the left of (11) is positive?” What is this “term”?

12. On p.7 in row 7 from below, there is $\sqrt{\Delta} \leq -\frac{ca^3}{3\alpha_1} + \frac{2}{3}\sqrt{\Delta_2}$. In reality, the inequality $\sqrt{\Delta} \leq -\frac{ca^2}{\alpha_1} + \frac{2c^2a^4}{3\alpha_1^2} + \frac{2}{3}\sqrt{\Delta_2}$ takes place.

13. The text in rows 4–6 from below on p.7 requires clarification. For example, why do the inequalities $\frac{ca^2}{3\alpha_1} + \frac{2}{3}\sqrt{\Delta_2} > 0$ and $-\frac{c^2a^4}{\alpha_1^2} + 6\beta d_1 < 0$ hold?

14. The expression for Jacobian $J(x_1, -\frac{ca^2}{2\alpha_1\beta})$ (p.8) is obtained from the expression for Jacobian $J(x, y)$ (p.6) by substitution $y = -\frac{ca^2}{2\alpha_1\beta}$. This is done with an error. As a consequence, the assertions on p.8 (before Theorem 3.4) are wrong.

15. On p. 8, in row 3 from below, a strange symbol \mathbb{R}_+^* is present.

Taking into account the remarks listed above, I recommend to decline the paper.