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On a nonlocal boundary value problem for the degenerating mixed type equation fractional derivative

Abstract: Main goal of this work is to study the existence and the uniqueness of solution of a non-local problem for the degenerating mixed type. Considering parabolic-hyperbolic equation involve the Caputo fractional derivative fractional order. The uniqueness of solution is proved by the method of integral energy using necessary properties of hypergeometric functions and integro-differential operators fractional order. The existence is proved by the method of integral equations.

Key words and phrases: Boundary value problem, degenerating equation,parabolic-hyperbolic type, Gauss hypergeometric function, Cauchy problem, existence and uniqueness of solution, a principle an extremume, method of integral equations, Caputo fractional derivative.

Introduction

Well known, that the theory fractional differential equations is one the much attended directions of DEs (see [1]-[4]). Moreover, the fractional calculus is widely applied to investigations of problems of partial differential equations and also of mixed type equation with degenerations [4]-[6]. In a series of papers [7]-[9], the authors considered some classes of boundary value problems for mixed type non-degenerating and degenerating differential equations involving Caputo and Riemann-Liouville fractional derivatives of order $0 < \alpha \leq 1$.

This work deals the existence and uniqueness of solution of the problem for the mixed type equation with two lines of degenerating which involve the Caputo fractional derivative.

Definition. Caputo fractional derivatives ${}_C D^{\alpha}_{ax} f$ and ${}_C D^{\alpha}_{xb} f$ of order $\alpha > 0$, $(\alpha \notin N \cup \{0\})$ are defined by [1.p.92]:

$$\left({}_C D^{\alpha}_{ax} f\right) x = \frac{1}{\Gamma(n-\alpha)} \int_a^x \frac{f^{(n)}(t)}{(x-t)^{\alpha-n+1}} dt, \quad n = [\alpha] + 1, \quad x > a \; ; \tag{1}$$

$$\left({}_C D^{\alpha}_{xb} f\right) x = \frac{(-1)^n}{\Gamma(n-\alpha)} \int_x^b \frac{f^{(n)}(t)}{(t-x)^{\alpha-n+1}} dt, \quad n = [\alpha] + 1, \;\; x < b \tag{2}$$

respectively.

From (1), (2), as a conclusion we will have: $k-1 < \alpha \leq k$, $k \in N$; consequently, while for $\alpha \in N \cup \{0\}$ we have

$$\left({}_C D^{0}_{ax} f\right) x = f(x), \left({}_C D^{0}_{xb} f\right) x = f(x), \left({}_C D^{n}_{ax} f\right) x = f^{(n)}(x) \; ;$$

$$\left({}_C D^{n}_{xb} f\right) x = (-1)^n f^{(n)}(x), \quad n \in N.$$

Gauss hyper geometric function $F(a, b, c, z)$ is defined in the unit desk as the sum of the hyper geometric series (see [1. p.27]):

$$F(a, b, c, z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{z^k}{k!}, \tag{3}$$

where
$|z| < 1, a,\ b \in \mathrm{C}, c \in \mathrm{C}\backslash Z_0^-$ and $(a)_0 = 1, \quad (a)_n = a(a+1)...(a+n-1) = \frac{\Gamma(a+n)}{\Gamma(a)}, n \in N$.
One such analytic continuation is given by Eyler integral representation:

$$F(a, b, c; z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 x^{b-1}(1-x)^{c-b-1}(1-zx)^{-a} dx,$$

$$0 < Reb < Rec, \quad \left|\arg(1-z)\right| < \pi. \tag{4}$$

The Gauss hypergeometric function $F(a, b, c, z)$ allows the following estimation:

$$F(a, b, c; z) \leq \begin{cases} c_1, & if \quad c-a-b > 0, \quad 0 \leq z \leq 1, \\ c_2(1-z)^{c-a-b}, & if \quad c-a-b < 0, \quad 0 < z < 1, \\ (1+|\ln(1-z)|)A_3, & if \quad c-a-b = 0, \end{cases} \tag{5}$$

$$F(a, 1-a, c, z) = (1-z)^{c-1} F\left(\frac{c-a}{2}, \frac{c+a-1}{2}, c, 4z(1-z)\right). \tag{6}$$

Generalized fractional integro-differential operators with Gauss hypergeometric function $F(a, b, c, z)$ defined for real $a,\ b,\ c$ and $x > 0$will be given by formulate:

$$F_{ox}\left[\begin{array}{cc} a, & b \\ c, & x^k \end{array}\right] f(x) = \frac{1}{\Gamma(c)} \int_0^x f(t)(x^k - t^k)^{c-1} F\left(a, b, c; \frac{x^k - t^k}{x^k}\right) kt^{t-1} dt,\ c > 0,\ k > 0. \tag{7}$$

The elementary definition of the Wright type function at $\alpha > \beta$, $\alpha > 0$ and for all $z \in \mathrm{C}$, is [10]

$$e^{\mu,\delta}_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \mu)\Gamma(\delta - \beta n)}. \tag{8}$$

If $\alpha = \mu = 1$, then owing to (3) from (8) we have:

$$e^{1,\delta}_{1,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^n}{n!\Gamma(\delta - \beta n)}. \tag{9}$$

Problem formulation and main functional relation

We consider equation:

$$0 = \begin{cases} u_{xx} - {}_C D^{\alpha}_{oy} u, & x > 0,\ \ y > 0, \\ (-y)^m u_{xx} - x^n u_{yy}, & x > 0,\ \ y < 0 \end{cases} \tag{10}$$

with operators (*see* (1)):

$${}_C D^{\alpha}_{oy} u = \frac{1}{\Gamma(1-\alpha)} \int_0^y (y-t)^{-\alpha} u_t(x, t) dt, \tag{11}$$

where $0 < \alpha < 1, \quad m = const > 0$.
Let's Ω domain, bounded with segments: $A_1A_2 = \{(x, y) : \ \ x = 0,\ 0 < y < h_2,\}$ $B_1B_2 = \{(x, y) : \ \ x = h_1,\ 0 < y < h_2\}$, $A_2B_2 = \{(x, y) : \ \ y = h_2,\ 0 < x < h_1\}$

at the $y>0$ and by the characteristics:

$$A_1C:\ \frac{1}{q}x^q-\frac{1}{p}(-y)^p=0,\quad B_1C:\ \frac{1}{q}x^q+\frac{1}{p}(-y)^p=1;$$

of equation (10) at $y<0$, where $A_1(0;0)$, $A_2(0;h_2)$, $B_1(h_1;0)$ $B_2(h_1;h_2)$ and $C\left(\left(\frac{q}{2}\right)^{1/q},-\left(\frac{p}{2}\right)^{1/p}\right)$. Here $2q=n+2$, $2p=m+2$, $h_1=q^{1/q}$, $h_2>0$, and that $m>n$.
Introduce designations: $\theta(x)=\left(\frac{x^q}{2}\right)^{1/q}-i\left(\frac{p}{q}\frac{x^q}{2}\right)^{1/p}$, $2\alpha_1=n/(n+2)$, $2\beta_1=m/(m+2)$,

$$0<\alpha_1<\beta_1<\frac{1}{2}, \tag{12}$$

$\Omega^+=\Omega\cap(y>0), \Omega^-=\Omega\cap(y<0), I_1=\{x:\ 0<x<h_1\}, I_2=\{y:0<y<h_2\}$.

For the equation (10), we consider the following problem:

Problem I. Find a solution $u(x,y)$ of equation (10) from the following class of functions:
$\Delta=\left\{u(x,y):\ u(x,y)\in C(\bar{\Omega})\cap C^2(\Omega^-),\quad u_{xx}\in C(\Omega^+),\quad {}_CD_{oy}^{\alpha}u\in C(\Omega^+)\right\}$
satisfies boundary conditions:

$$u(x,y)\Big|_{A_1A_2}=\varphi_1(y), 0\le y\le h_2, \tag{13}$$

$$u(x,y)\Big|_{B_1B_2}=\varphi_2(y), h_1\le y\le h_2, \tag{14}$$

$$\frac{d}{d\left(x^{2q}\right)}\left(x^{2q}\right)^{\frac{1-\alpha_1-\beta_1}{2}}F_{ox}\left[\begin{matrix}\frac{\alpha_1+\beta_1-1}{2}, & \frac{\alpha_1+\beta_1}{2}\\ \beta_1, & x^{2q}\end{matrix}\right]\left(x^{2q}\right)^{\frac{2\alpha_1-1}{2}}u\left[\theta(x)\right]=$$

$$=a(x)u_y(x,0)+b(x),\qquad 0<x<h_1, \tag{15}$$

and gluing condition:

$$\lim_{y\to+0}y^{1-\alpha}u_y(x,+0)=u_y(x,-0),(x,0)\in A_1B_1; \tag{16}$$

where $\varphi_1(y)$, $\varphi_2(y)$, $\tilde{a}(x)=a(x^{1/2q}), \tilde{b}(x)=b(x^{1/2q})$ are given functions, and that

$$\varphi_1(y),\ \varphi_2(y)\in C\left(\bar{I}_2\right)\cap C^1\left(I_2\right), a(x)\in C(\bar{I}_1)\cap C^2(I_1), b(x)\in C^2(I_1),$$

In fact, that functional relation between $\tau(x)$ and $\nu(x)$ transferred from the parabolic part Ω^+ (hyperbolic part Ω^- to the line $y=0$) is played important role on the proved unique and existence of solution.
It is well know, that the solution of the Cauchy problem for the equation (10) in domain Ω^- satisfies conditions

$$u(x,-0)=\tau^-(x),\quad 0\le x\le 1u_y(x,-0)=\nu^-(x),\quad 0<x<1, \tag{17}$$

presented on the form [11, 12]:

$$u(x,y)=\frac{\Gamma\left(\frac{1}{2}+\beta_1\right)}{\sqrt{\pi}\Gamma(\beta_1)}2^{2\beta_1-1}\left(\frac{1}{q}x^q\right)^{-\alpha_1}\int_0^1\left[\frac{1}{p}(-y)^p(2z-1)+\frac{1}{q}x^q\right]^{\alpha_1}\times$$

$$\times [z(1-z)]^{\beta_1-1}\,\tau^-\left\{\left[\frac{q}{p}(-y)^p(2z-1)+x^q\right]^{\frac{1}{q}}\right\} F(\alpha_1,1-\alpha_1,\beta_1,\rho)dz-$$

$$-\frac{(2p)^{1-2\beta_1}\Gamma\left(\frac{3}{2}+\beta_1\right)}{\sqrt{\pi}\Gamma(1-\beta_1)}\left(\frac{1}{p}(-y)^p\right)^{1-2\beta_1}\left(\frac{1}{q}x^q\right)^{-\alpha_1}\int_0^1\left[\frac{1}{p}(-y)^p(2z-1)+\frac{1}{q}x^q\right]^{\alpha_1}\times$$

$$\times [z(1-z)]^{-\beta_1}\,\nu^-\left\{\left[\frac{q}{p}(-y)^p\cdot(2z-1)+x^q\right]^{\frac{1}{q}}\right\} F(\alpha_1,1-\alpha_1,\beta_1,\rho)dz, \qquad (18)$$

where $\rho=\frac{q(-y)^{\frac{1}{p}}z(1-z)}{p^2x^q\left[\frac{1}{p}(-y)^p(2z-1)+\frac{1}{q}x^q\right]}$.

Further, using formulates (6) and (7), from (18) we will find

$$u[\theta(x)]=\gamma_1(x^{2q})^{\frac{2-\alpha_1-3\beta_1}{2}}F_{0x}\left[\begin{array}{cc}\frac{\beta_1-\alpha_1}{2}, & \frac{\alpha_1+\beta_1-1}{2}\\ \beta_1, & x^{2q}\end{array}\right](x^{2q})^{\frac{\alpha_1+\beta_1-2}{2}}\tau^-(x)-$$

$$-\gamma_2(x^{2q})^{\frac{\beta_1-\alpha_1}{2}}F_{0x}\left[\begin{array}{cc}\frac{1-\beta_1-\alpha_1}{2}, & \frac{\alpha_1-\beta_1}{2}\\ 1-\beta_1, & x^{2q}\end{array}\right](x^{2q})^{\frac{\alpha_1-\beta_1-1}{2}}\nu^-(x), \qquad (x,0)\in I_1, \qquad (19)$$

where $\gamma_1=\frac{\Gamma(2\beta_1)}{\Gamma(\beta_1)}2^{\alpha_1-\beta_1}$, $\quad \gamma_2=\frac{2^{\alpha_1+3\beta_1-2}\Gamma(2-2\beta_1)}{\Gamma(1-\beta_1)}\left(\frac{p}{q}\right)^{1-2\beta_1}$.

Based on the condition (14), from (19) we obtain functional relation between$\tau^-(x)$ and $\nu^-(x)$ transferred from hyperbolic domain Ω^- on the line $y=0$:

$$\bar{a}(x)\nu^-(x)=\gamma_1\left(x^{2q}\right)^{\frac{1-2\alpha_1}{2}}\frac{d}{dx^{2q}}\left(x^{2q}\right)^{\frac{1-2\beta_1}{2}}F_{0x}\left[\begin{array}{cc}\alpha_1+\beta_1, & \frac{2\beta_1-1}{2}\\ 2\beta_1, & x\end{array}\right]\left(x^{2q}\right)^{\frac{2\alpha_1-1}{2}}\tau^-(x)-$$

$$-\left(x^{2q}\right)^{\frac{1-\alpha_1+\beta_1}{2}}b(x), \quad 0<x<h_1, \qquad (20)$$

where $\bar{a}(x)\equiv\left(x^{2q}\right)^{\frac{1-\alpha_1+\beta_1}{2}}a(x)+\gamma_2$.

On the other hand, considering designations (17) and $\lim\limits_{y\to+0}y^{1-\alpha}u_y(x,y)=\nu^+(x)$, $0<x<h_1$ from gluing condition (16) we have

$$\nu^+(x)=\nu^-(x). \qquad (21)$$

For further supposes, from Eq.(10) at $y\to+0$ considering (11), (21) and

$$\lim_{y\to0}D_{0y}^{\alpha-1}f(y)=\Gamma(\alpha)\lim_{y\to0}y^{1-\alpha}f(y)$$

we get:

$$\tau''(x)-\Gamma(\alpha)\nu^+(x)=0. \qquad (22)$$

The uniqueness of solution.

Theorem 1. If satisfy conditions $0<\alpha<1$, (12) and

$$\bar{a}(x)\equiv\left(x^{2q}\right)^{\frac{1-\alpha_1+\beta_1}{2}}a(x)+\gamma_2>0, \quad \forall(x,0)\in\bar{I}_1,$$

then, the solution is unique.

Proof . As usual, we consider corresponding homogeneous problem
$(\varphi_1(y) \equiv \varphi_2(y) \equiv 0)$ and prove that $u(x,y) \equiv 0$. With this aim we multiply to $\tau(x)$ equation (22) and integrate from 0 to h_1 :

$$\Gamma(\alpha)\int_0^{h_1}\tau(x)\nu^+(x)dx = \int_0^{h_1}\tau''(x)\tau(x)dx. \tag{23}$$

Integrating by part and using the relations $\tau(0) = \tau(h_1) = 0$, owing to (21)we obtain

$$\int_0^{h_1}\tau(x)\nu^-(x)dx = \int_0^{h_1}\tau(x)\nu^+(x)dx = -\int_0^{h_1}\left(\tau'(x)\right)^2 dx \le 0. \tag{24}$$

Now, we prove that $\int_0^{h_1}\tau(x)\nu^-(x)dx \ge 0$.
At first, using by formulate (7) we make some simplifications in (20):

$$\nu^-(x) = \frac{\gamma_1\left(x^{2q}\right)^{\frac{1-2\alpha_1}{2}}}{\bar{a}(x)\Gamma(2\beta_1)}\frac{d}{dx^{2q}}\left(x^{2q}\right)^{\frac{1-2\beta_1}{2}}\int_0^x\left(x^{2q}-t^{2q}\right)^{2\beta_1-1}\left(t^{2q}\right)^{\frac{2\alpha_1-1}{2}}\times$$
$$\times\tau^-(t)F\left(\alpha_1+\beta_1,\frac{2\beta_1-1}{2},2\beta_1,\frac{x^{2q}-t^{2q}}{x^{2q}}\right)dt^{2q} - \frac{\left(x^{2q}\right)^{\frac{1-\alpha_1+\beta_1}{2}}b(x)}{\bar{a}(x)}, \tag{25}$$

Entering replacement $t = xz$ and after some simplifications we have:

$$\nu^-(x) = \frac{2q\gamma_1\left(\alpha_1+\beta_1\right)\left(x^{2q}\right)^{\frac{2\beta_1-1}{2}}}{\bar{a}(x)\Gamma(2\beta_1)}\int_0^1\left(1-z^{2q}\right)^{2\beta_1-1}\left(z^{2q}\right)^{\frac{2\alpha_1+1}{2}}\times$$
$$\times\tau^-(xz)F\left(\beta_1-\alpha_1,\frac{2\beta_1+1}{2},2\beta_1,1-z^{2q}\right)dz + \frac{2q\,\gamma_1\left(x^{2q}\right)^{\frac{2\beta_1+1}{2}}}{\bar{a}(x)\Gamma(2\beta_1)}\times$$
$$\times\int_0^1\left(1-z^{2q}\right)^{2\beta_1-1}z^{2q}\,\tau'^-(xz)F\left(\beta_1-\alpha_1,\frac{2\beta_1+1}{2},2\beta_1,1-z^{2q}\right)dz - \frac{\left(x^{2q}\right)^{\frac{1-\alpha_1+\beta_1}{2}}b(x)}{\bar{a}(x)}.$$

Consequently, using inverse replacements $s = x\,z$ we can receive

$$\nu^-(x) = \frac{2q\gamma_1\left(\alpha_1+\beta_1\right)}{\bar{a}(x)\Gamma(2\beta_1)}\left(x^{2q}\right)^{-\alpha_1-\beta_1}\int_0^x\left(x^{2q}-s^{2q}\right)^{2\beta_1-1}\left(s^{2q}\right)^{2\alpha_1}\tau^-(s)\times$$
$$\times F\left(\alpha_1+\beta_1,\frac{2\beta_1-1}{2},2\beta_1,\frac{x^{2q}-s^{2q}}{x^{2q}}\right)ds + \frac{2q\gamma_1}{\bar{a}(x)\Gamma(2\beta_1)}\left(x^{2q}\right)^{\frac{1}{2}-\beta_1}\int_0^x\left(s^{2q}\right)^{\alpha_1+\frac{1}{2}}\times$$
$$\times\left(x^{2q}-s^{2q}\right)^{2\beta_1-1}\tau'^-(s)F\left(\alpha_1+\beta_1,\frac{2\beta_1-1}{2},2\beta_1,\frac{x^{2q}-s^{2q}}{x^{2q}}\right)ds - \frac{\left(x^{2q}\right)^{\frac{1-\alpha_1+\beta_1}{2}}b(x)}{\bar{a}(x)}. \tag{26}$$

There holds the following preliminary assertion (see [14]).

Lemma. If a function $\tau(x)$ has a positive maximum (respectively a negative minimum) at the point $x = x_0 \in (0, h_1)$, then $\nu^-(x_0) > 0$ (respectively).
Proof. Let's a function $\tau(x)$ has a positive maximum at the point and $b(x) \equiv 0$, then from (26) we have:

$$\nu^-(x_0) = \frac{2q\gamma_1(\alpha_1+\beta_1)}{\bar{a}(x)\Gamma(2\beta_1)}\left(x_0^{2q}\right)^{-\alpha_1-\beta_1}\int_0^{x_0}\tau'^-(t)dt\int_t^{x_0}\left(x_0^{2q}-s^{2q}\right)^{2\beta_1-1}\left(s^{2q}\right)^{2\alpha_1}\times$$

$$\times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{x_0^{2q}-s^{2q}}{x_0^{2q}}\right)ds + \frac{2q\gamma_1}{\bar{a}(x)\Gamma(2\beta_1)}\left(x_0^{2q}\right)^{\frac{1}{2}-\beta_1}\times$$

$$\times\int_0^x \tau'^-(s)\left(s^{2q}\right)^{\alpha_1+\frac{1}{2}}\left(x_0^{2q}-s^{2q}\right)^{2\beta_1-1}F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{x_0^{2q}-s^{2q}}{x_0^{2q}}\right)ds.$$

Due to $\gamma_1 > 0$, $\bar{a}(x) > 0$, $\Gamma(2\beta_1) > 0$, $0 \le \frac{x_0^{2q}-s^{2q}}{x_0^{2q}} \le 1$, $\int_0^{x_0}\tilde{\tau}'(s)ds = \int_0^{x_0}\lim\limits_{x_0\to s}\frac{\tilde{\tau}(x_0)-\tilde{\tau}(s)}{x_0-s}ds > 0$ and taking (12), (5) into account, from here we deduce that $\nu^-(x_0) > 0$.
Similarly, we can prove that on the point of negative minimum $\nu^-(x_0) < 0$. ***Lemma is proved.***

Based on the Lemma, we can conclude that, $\int_0^{h_1}\tau(x)\nu^-(x)dx \ge 0$, consequently from (24) we will get $\nu^-(x) \equiv \tau(x) \equiv 0$. Hence, based on the solution of the first boundary problem for the Eq. (10) [7],[15] owing to account (13) and (14)we will get $u(x, y) \equiv 0$ in $\overline{\Omega}^+$, similarly, based on the solution (18) we obtain $u(x, y) \equiv 0$ in closed domain $\bar{\Omega}^-$.
The existence of solution of the Problem I.

Theorem 2. If satisfies all conditions of the **Theorem 1** and

$$\varphi_1(y),\ \varphi_2(y) \in C\left(\bar{I}_2\right) \cap C^1\left(I_2\right), a(x) \in C(\bar{I}_1) \cap C^2(I_1), b(x) \in C^2(I_1), \tag{27}$$

than the solution of the investigating problem is exist.
Proof. Taking (21) into account from Eq. (22) we will obtain

$$\tau''(x) = f(x), \tag{28}$$

where

$$f(x) = \Gamma(\alpha)\nu^-(x). \tag{29}$$

Solution of the equation (28) together with conditions $\tau(0) = \varphi_1(0)$, $\tau(h_1) = \varphi_2(0)$ has a form:

$$\tau(x) = \int_0^x (x-t)f(t)dt - x\int_0^1 (1-t)f(t)dt + \varphi_2(0)(1-x) + x\varphi_1(0),$$

consequently, we can find:

$$\tau'(x) = \int_0^x f(t)dt - \int_0^1 (1-t)f(t)dt + \varphi_1(0) - \varphi_2(0). \tag{30}$$

Further, considering (29) from (30), after some simplifications we will get

$$\tau'(x) = \Gamma(\alpha)\int_0^x \nu(t)dt - \Gamma(\alpha)\int_0^1 (1-t)\nu(t)dt + \varphi_1(0) - \varphi_2(0). \qquad (31)$$

Substituting (26) into (31)we have:

$$\begin{aligned}
\tau'(x) = {} & \frac{2q\gamma_1(\alpha_1+\beta_1)\Gamma(\alpha)}{\bar{a}(x)\Gamma(2\beta_1)}\int_0^x \left(t^{2q}\right)^{-\alpha_1-\beta_1} dt \int_0^t \left(t^{2q}-s^{2q}\right)^{2\beta_1-1}\left(s^{2q}\right)^{2\alpha_1} \times \\
& \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds \int_0^s \tau'(z)dz + \\
& + \frac{2q\gamma_1\Gamma(\alpha)}{\bar{a}(x)\Gamma(2\beta_1)}\int_0^x \left(t^{2q}\right)^{\frac{1-2\beta_1}{2}} dt \int_0^t \left(t^{2q}-s^{2q}\right)^{2\beta_1-1}\left(s^{2q}\right)^{\frac{2\alpha_1+1}{2}} \tau'(s) \times \\
& \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds - \\
& - \frac{2q\gamma_1(\alpha_1+\beta_1)\Gamma(\alpha)}{\bar{a}(x)\Gamma(2\beta_1)}\int_0^1 (1-t)\left(t^{2q}\right)^{-\alpha_1-\beta_1} dt \int_0^t \left(t^{2q}-s^{2q}\right)^{2\beta_1-1}\left(s^{2q}\right)^{2\alpha_1} \times \\
& \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds \int_0^s \tau'(z)dz - \\
& - \frac{2q\gamma_1\Gamma(\alpha)}{\bar{a}(x)\Gamma(2\beta_1)}\int_0^1 (1-t)\left(t^{2q}\right)^{\frac{1-2\beta_1}{2}} dt \int_0^t \left(t^{2q}-s^{2q}\right)^{2\beta_1-1}\left(s^{2q}\right)^{\frac{2\alpha_1+1}{2}} \tau'(s) \times \\
& \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds + F(x), \qquad (32)
\end{aligned}$$

where

$$\begin{aligned}
F(x) = {} & \frac{2q\gamma_1(\alpha_1+\beta_1)\Gamma(\alpha)}{\bar{a}(x)\Gamma(2\beta_1)}\int_0^x \left(t^{2q}\right)^{-\alpha_1-\beta_1} dt \int_0^t \left(t^{2q}-s^{2q}\right)^{2\beta_1-1}\left(s^{2q}\right)^{2\alpha_1} \varphi_2(0) \times \\
& \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds - \frac{2q(\alpha_1+\beta_1)\gamma_1\Gamma(\alpha)}{\bar{a}(x)\Gamma(2\beta_1)} \times \\
& \times \int_0^1 (1-t)\left(t^{2q}\right)^{-\alpha_1-\beta_1} dt \int_0^t \left(t^{2q}-s^{2q}\right)^{2\beta_1-1}\left(s^{2q}\right)^{2\alpha_1} \varphi_2(0) \times \\
& \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds - \frac{\Gamma(\alpha)}{\bar{a}(x)}\int_0^x \left(t^{2q}\right)^{\frac{1-\alpha_1+\beta_1}{2}} b(t)dt + \\
& + \frac{\Gamma(\alpha)}{\bar{a}(x)}\int_0^1 (1-t)\left(t^{2q}\right)^{\frac{1-\alpha_1+\beta_1}{2}} b(t)dt + \varphi_1(0) - \varphi_2(0). \qquad (33)
\end{aligned}$$

Changing the order of integration in (32), totally we have integral equation

$$\tau'(x) = \int_0^1 K(x,z)\tau'(z)dz + F(x). \qquad (34)$$

Here

$$K(x,z) = \begin{cases} K_1(x,z), & 0 \le z \le x, \\ K_2(x,z), & x \le z \le 1, \end{cases} \tag{35}$$

$$K_1(x,z) = \frac{2q\gamma_1(\alpha_1+\beta_1)\Gamma(\alpha)}{\bar{a}(x)\Gamma(2\beta_1)} \int_z^x \left(t^{2q}\right)^{-\alpha_1-\beta_1} dt \int_z^t \left(t^{2q}-s^{2q}\right)^{2\beta_1-1} \left(s^{2q}\right)^{2\alpha_1} \times$$
$$\times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds - \frac{2q\gamma_1(\alpha_1+\beta_1)\Gamma(\alpha)}{\bar{a}(x)\Gamma(2\beta_1)} \times$$
$$\times \int_z^1 (1-t)\left(t^{2q}\right)^{-\alpha_1-\beta_1} dt \int_z^t \left(t^{2q}-s^{2q}\right)^{2\beta_1-1} \left(s^{2q}\right)^{2\alpha_1} \times$$
$$\times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds + \frac{2q\gamma_1\Gamma(\alpha)}{\bar{a}(x)\Gamma(2\beta_1)} \left(z^{2q}\right)^{\frac{2\alpha_1+1}{2}} \times$$
$$\times \int_z^x \left(t^{2q}\right)^{\frac{1-2\beta_1}{2}} \left(t^{2q}-z^{2q}\right)^{2\beta_1-1} F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-z^{2q}}{t^{2q}}\right) dt - \frac{2q\gamma_1\Gamma(\alpha)}{\bar{a}(x)\Gamma(2\beta_1)} \times$$
$$\times \left(z^{2q}\right)^{\frac{2\alpha_1+1}{2}} \int_z^1 (1-t)\left(t^{2q}\right)^{\frac{1-2\beta_1}{2}} \left(t^{2q}-z^{2q}\right)^{2\beta_1-1} F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-z^{2q}}{t^{2q}}\right) dt, \tag{36}$$

$$K_2(x,z) = \frac{2q\gamma_1\Gamma(\alpha)(\alpha_1+\beta_1)}{\bar{a}(x)\Gamma(2\beta_1)} \int_z^1 (1-t)\left(t^{2q}\right)^{-\alpha_1-\beta_1} dt \int_z^t \left(t^{2q}-s^{2q}\right)^{2\beta_1-1} \left(s^{2q}\right)^{2\alpha_1} \times$$
$$\times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds - \frac{2q\gamma_1\Gamma(\alpha)}{\bar{a}(x)\Gamma(2\beta_1)} \left(z^{2q}\right)^{\frac{2\alpha_1+1}{2}} \times$$
$$\times \int_z^1 (1-t)\left(t^{2q}\right)^{\frac{1-2\beta_1}{2}} \left(t^{2q}-z^{2q}\right)^{2\beta_1-1} F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-z^{2q}}{t^{2q}}\right) dt. \tag{37}$$

Due to properties of hyper geometric function (5) from (37) we will get

$$|K_1(x,z)| \le \left| \int_z^x \left(t^{2q}\right)^{-\alpha_1-\beta_1} dt \left| \int_z^t \left(t^{2q}-s^{2q}\right)^{2\beta_1-1} \left(s^{2q}\right)^{2\alpha_1} s^{1-2q} ds^{2q} \right|\right|. \tag{38}$$

Hence, due to class of given functions (38) considering (36) and (37) from (33) and (35) respectively we will receive $|K(x,z)| \le const$ for all $0 \le x,\ z \le 1$, $|F(x)| \le const$, $0 \le x \le 1$.

Since kernel $K(x,z)$ is continuous and function in right-side $F(x)$ is continuously differentiable, solution of integral equation (34) we can write via resolvent-kernel:

$$\tau'(x) = F(x) - \int_0^1 \Re(x,z) F(z) dz, \tag{39}$$

where $\Re(x,z)$ is the resolvent-kernel of $K(x,z)$.

Unknown functions $\nu^-(x)$ we will in accordingly from (26).

Solution of the Problem I in the domain Ω^+ we will write as follows [15], [7]:

$$u(x,y) = \int_0^y G_\xi(x,y,0,\eta)\psi(\eta)d\eta - \int_0^y G_\xi(x,y,1,\eta)\varphi(\eta)d\eta + \int_0^1 G_0(x-\xi,y)\tau(\xi)d\xi,$$

here

$$G_0(x-\xi,y) = \frac{1}{\Gamma(1-\alpha)}\int_0^y \eta^{-\alpha}G(x,y,\xi,\eta)d\eta,$$

$$G(x,y,\xi,\eta) = \frac{(y-\eta)^{\frac{\alpha}{2}-1}}{2}\sum_{n=-\infty}^{\infty}\left[e^{1,\alpha/2}_{1,\alpha/2}\left(-\frac{|x-\xi+2n|}{(y-\eta)^{\alpha/2}}\right) - e^{1,\alpha/2}_{1,\alpha/2}\left(-\frac{|x+\xi+2n|}{(y-\eta)^{\alpha/2}}\right)\right].$$

Is the Green's function of the first boundary problem Eq. (10) in the domain Ω^+ with the Riemanne-Liouville fractional differential operator instead of the Caputo ones [15],

$$e^{1,\delta}_{1,\delta}(z) = \sum_{n=0}^{\infty}\frac{z^n}{n!\Gamma(\delta-\delta n)}$$

is the Wright type function [10].

Solution of the Problem I in the domain Ω^- will be found by the formulate (18). Hence, the **Theorem2** is proved.

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