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PROBLEM OF MULTIPLE CAPTURE OF GIVEN NUMBER OF EVADERS IN RECURRENT DIFFERENTIAL GAMES

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ABSTRACT. The problem of pursuit by a group of pursuers of a group of evaders with equal opportunities for all participants and geometric restrictions on the control of players is considered. The evaders use program strategies, each pursuer catches no more than one evader. The goal of the pursuers is to catch a given number of evaders, and each evader needs to be caught no less than a certain number of pursuers. In this paper, sufficient conditions are obtained for multiple capture of a given number of evaders.

Keywords: differential game, pursuer, evader, recurrent function.

1. Introduction

Differential games of two players, first considered in the book of Isaacks [1], now present the wide field of researches. The natural generalization of pursuit-evasion differential games of two persons are games with a group of pursuers and one or several evaders [2, 3, 4, 5]. These games are interesting from the theoretical perspective, because they cannot be solved by the theory for two-players games. The union of pursuers reachability sets and the target sets union represent sets, which are non-convex and furthermore are not connected. There are some applications of these games to problems of vehicles motion avoidance of ship collisions, and others.

Multiple capture of one evader in the simple pursuit problem was investigated in papers [6, 7, 8], and in work [8] in the discrete statement. In works [9, 10], the

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problem of multiple capture of one evader in the Pontryagin's example is presented, and in works [11, 12] in linear differential games.

The task of capturing a given number of evaders in a simple pursuit problem, provided that evaders use program strategies and each pursuer catches no more than one evader, is represented in article [13], where necessary and sufficient conditions for the tractability of the pursuit problem were obtained.

In this paper, previously considered separately the problems of multiple capture and capture of a given number of evaders are combined into one task. The aim of the group of pursuers is to catch at least q evaders, and each evader must be caught by at least r pursuers. Assuming that evaders use program strategies, and each evader must be caught by at least r pursuers, sufficient conditions for the solvability of the pursuit problem are obtained.

2. Statement of Problem

In the space $\mathbb{R}^k$ $(k \geqslant 2)$, a differential game $\Gamma(n,m)$ of $n+m$ objects with n pursuers $P_1,\ldots,P_n$ and m evaders $E_1,\ldots,E_m$ is considered. The game $\Gamma(n,m)$ is described by a system of the form

$$\dot{z}_{ij} = A(t)z_{ij} + u_i - v_j,\ u_i, v_j \in V,\ z_{ij}(t_0) = z_{ij}^0, \tag{1}$$

where $z_{ij}, u_i, v_j \in \mathbb{R}^k$, $i \in I = \{1,\ldots,n\}$, $j \in J = \{1,\ldots,m\}$, V is a strictly convex compact set in $\mathbb{R}^k$, $A(t)$ is a continuous matrix function of order k on semi-axis $[t_0,\infty)$. We believe that $z_{ij}^0 \notin M_{ij}$, where M_{ij} are given convex compact sets.

Denote by $\Phi(t)$ the fundamental matrix of the system

$$\dot{\omega}(t) = A(t)\omega(t),\ \omega(t_0) = E.$$

Int$X$ denotes interior of the set X, co$X$ denotes convex hull of the set X. Let K be a finite subset of the set of natural numbers,

$$\lambda(h_{ij}, v) = \begin{cases} \sup\{\lambda \geqslant 0 : -\lambda(h_{ij} - M_{ij}) \cap (V - v) \neq \emptyset\},\ \text{if } h_{ij} \notin M_{ij}, \\ 0,\ \text{if } h_{ij} \in M_{ij}, \end{cases}$$

$$F_{ij}(t) = \int_{t_0}^{t} \lambda(v(s), \Phi(s)z_{ij}^0)ds,\ D_\varepsilon(a) = \{z \in R^k \mid \|z - a\| \leqslant \varepsilon\},$$

$$\Omega_l(K) = \{(i_1,\ldots,i_l) : i_p \in K, p = 1,\ldots,l \text{ and pairwise different}\},$$

Definition 1. *There is a r-multiple capture of the evader E_β in the game $\Gamma(n,m)$ if for each $\varepsilon > 0$ there exists an instant $T > t_0$ such that for any admissible control $v_\beta(t)$, $t \in [t_0,\infty)$ of the evader E_β, there exist admissible controls of the pursuers $u_i(t, z_{ij}^0, v_j(t),\ t \in [t_0,\infty))$, the set $\Lambda \in \Omega_I(r)$, and instants $\tau_l \in [t_0, T]$, $l \in \Lambda$, such that $z_{l\beta}(\tau_l) \in M_{l\beta} + D_\varepsilon(0)$ for all $l \in \Lambda$.*

Definition 2. *There is a r-multiple capture (at $r = 1$ capture) of at least q evaders in the game $\Gamma(n,m)$, if for each $\varepsilon > 0$ there exists $T > t_0$ such that for any set of admissible controls of the evaders v_j, $t \in [t_0,\infty)$, $j \in J$, there are admissible controls of the pursuers $u_i(t) = u_i(t, z_{ij}^0, v_j(s),\ s \in [t_0,\infty),\ j \in J)$ such that have the following property: there are sets*

$$M \subset J,\ |M| = q, \{N_\alpha,\ \alpha \in M\},\ N_\alpha \subset I,\ |N_\alpha| = r \textit{ for all } \alpha \in M,$$
$$N_\alpha \cap N_\beta = \emptyset \textit{ for all } \alpha \neq \beta,$$

such that the group of pursuers $\{P_\alpha, \alpha \in N_\beta\}$ not later than the instant T carries out the r-multiple capture of the evader E_β, and if the pursuer P_α catches the evader E_β, then the other evaders are considered not caught by it.

Definition 3. [14] *The function $f : \mathbb{R}^1 \to \mathbb{R}^k$ is called recurrent (by V.I. Zubov) if for any $\varepsilon > 0$ there exists $T(\varepsilon) > 0$ such that for any $a, t \in \mathbb{R}^1$, there exists $\tau(t) \in [a, a + T(\varepsilon)]$, for which it is true that*

$$\|f(t + \tau(t)) - f(t)\| < \varepsilon.$$

The function $f : [t_0, \infty[\to \mathbb{R}^k$ is called recurrent on $[t_0, \infty[$ if there exists a recurrent function $F : \mathbb{R}^1 \to \mathbb{R}^k$ such that $f(t) = F(t)$ for all $t \in [t_0, \infty[$.

Assumption 1. The fundamental matrix Φ is a recurrent function, and its derivative is uniformly bounded on $[t_0, \infty)$.

3. Multiple Capture of One Evader

Let $m = 1$.

Assumption 2. For all $i \in I$ $z_{i1} \notin M_{i1}$ and

$$\delta = \min_{v \in V} \max_{\Lambda \in \Omega_r(I)} \min_{\alpha \in \Lambda} \lambda(z_{\alpha 1}, v) > 0.$$

Lemma 1. *Suppose that Assumption 2 holds. Then there $\varepsilon_0 > 0$ such that for all $i \in I$, $h_{i1} \in D_{2\varepsilon_0}(z^0_{i1})$ exists*

(1) $h_{i1} \notin M_{i1}$;
(2) $\min_{v \in V} \max_{\Lambda \in \Omega_r(I)} \min_{\alpha \in \Lambda} \lambda(h_{\alpha 1}, v) > 0.$

The assertions of the lemma follow from the continuity of the function $\lambda(h_{i1}, v)$. In the future, we assume that ε_0 is chosen so that Lemma 1 holds.

Lemma 2. *Suppose that Assumption 1, 2 holds. Then there exists an instant $T > t_0$ such that for any admissible function $v(\cdot)$ there exist a set $\Lambda \in \Omega_r(I)$, such that $F_{\alpha 1}(T) \geqslant 1$ for all $\alpha \in \Lambda$.*

Proof. Denote by $\mu(A)$ the Lebesgue's measure of the set A,

$$\Delta = \{t \geqslant t_0 \mid \Phi(t) z^0_{i1} \in D_{2\varepsilon_0}(z^0_{i1}) \text{ for all } i \in I\},$$

$$d = \max_i \|z^0_{i1}\|, \quad M = \sup_{t \geqslant t_0} \|\dot{\Phi}(t)\|.$$

Since Φ is a recurrent function, there exists $\tau > 0$, such that for all $s = 1, 2, \dots$ there is an instant $t_s \in [t_0 + (s-1)\tau, t_0 + s\tau]$, for which

$$\|\Phi(t_s) - \Phi(t_0)\| < \frac{\varepsilon_0}{d}.$$

Then

$$\|\Phi(t_s) z^0_{i1} - \Phi(t_0) z^0_{i1}\| < \varepsilon_0.$$

Consequently, $\Phi(t_s) z^0_{i1} \in D_{\varepsilon_0}(z^0_{i1})$ for all $i \in I, s = 1, 2, \dots$ Let

$$\Delta_s = \{t \mid t \in [t_s, t_{s+1}),\ \Phi(t) z^0_{i1} \in D_{2\varepsilon_0}(z^0_{i1}) \text{ for all } i \in I\}.$$

From the mean value theorem, it follows that for any τ_1, τ_2, $i \in I$ the inequality

$$\|\Phi(\tau_1) z^0_{i1} - \Phi(\tau_2) z^0_{i1}\| \leqslant Md|\tau_1 - \tau_2|.$$

holds. Therefore, if

$$\|\Phi(\tau_1) z^0_{i1} - \Phi(\tau_2) z^0_{i1}\| \geqslant \varepsilon_0,$$

then $|\tau_1-\tau_2| \geqslant \tau_0 = \varepsilon_0/(Md)$. So $[t_s, t_s+\tau_0] \subset \Delta_s$ for all $s = 1, 2, \dots$ Consequently, $\mu(\Delta) = +\infty$. By Lemma 1, for any

$$h = (h_1, \dots, h_n) \in D = D_{2\varepsilon_0}(z_{11}^0) \times \cdots \times D_{2\varepsilon_0}(z_{n1}^0)$$

the inequality

$$\rho(h) = \min_{v\in V} \max_{\Lambda\in\Omega_r(I)} \min_{s\in\Lambda} \lambda(h_s, v) > 0.$$

holds. Since the function ρ is continuous on D, then

$$\delta = \min_{h\in D} \min_{v\in V} \max_{\Lambda\in\Omega_r(I)} \min_{s\in\Lambda} \lambda(h_s, v) > 0.$$

Further

$$\max_{\Lambda\in\Omega_r(I)} \min_{s\in\Lambda} F_{s1}(t) = \max_{\Lambda\in\Omega_r(I)} \min_{s\in\Lambda} \int_{t_0}^{t} \lambda(\Phi(t) z_{s1}^0, v(t))dt$$

$$\geqslant \max_{\Lambda\in\Omega_r(I)} \min_{s\in\Lambda} \int_{[t_0,t]\cap\Delta} \lambda(\Phi(t) z_{s1}^0, v(t))dt$$

$$\geqslant \frac{1}{C_n^r} \sum_{\Lambda\in\Omega_r(I)} \Big(\min_{s\in\Lambda} \int_{[t_0,t]\cap\Delta} \lambda(\Phi(t) z_{s1}^0, v(t))dt\Big) \geqslant \frac{\delta}{C_n^r} \mu([t_0, t] \cap \Delta).$$

Since $\mu(\Delta) = +\infty$, then $\lim_{t\to\infty} \mu([t_0, t] \cap \Delta) = +\infty$. Therefore, for the instant T from the condition

$$\frac{\delta}{C_n^r} \mu([t_0, T] \cap \Delta) \geqslant 1$$

and some $\Lambda \in \Omega_r(I)$ we obtain the inequality $F_{s1}(T) \geqslant 1$ for all $s \in \Lambda$. The lemma is proved. □

Let

$$T_0 = \min\{t > t_0 : \inf_{v(\cdot)} \max_{\Lambda\in\Omega_r(I)} \min_{\alpha\in\Lambda} F_{\alpha 1}(t) \geqslant 1\}.$$

By Lemma 2, $T_0 < \infty$.

Theorem 1. *Suppose that Assumption 1, 2 holds. Then in the game* $\Gamma(n, 1)$, *r-multiple capture occurs.*

Proof. By the Cauchy's formula, the solution of problem (1) for any admissible control has the form

$$z_{i1}(t) = \Phi(t)\Big(z_{i1}^0 + \int_{t_0}^{t} \Phi^{-1}(s)(u_s(s) - v(s))ds\Big). \tag{2}$$

Let $v(t), t \in [t_0, T_0]$ be an arbitrary admissible control of the evader E_1. It follows from the definition of the instant T_0 that there exists an instant $\tau \in [t_0, T_0]$, being the root of the function

$$G(t) = 1 - \max_{\Lambda\in\Omega_r(I)} \min_{\alpha\in\Lambda} \int_{t_0}^{t} \lambda(\Phi(s) z_{\alpha 1}^0, v(s))ds,$$

and the set $\Lambda_0 \in \Omega_r(I)$ such that

$$1 - \int_{t_0}^{t} \lambda(\Phi(s)z^0_{\alpha 1}, v(s))ds \leqslant 0 \text{ for all } \alpha \in \Lambda_0.$$

Let $t_\alpha \in [t_0, \infty)$ be the least root of the function $1 - F_{\alpha 1}(t)$, $\alpha \in \Lambda_0$. Note that $t_\alpha \in [t_0, \tau]$ for all $\alpha \in \Lambda_0$. Let $C = \max_{i, m_i \in M_{i1}} \|m_i\|$, $\varepsilon > 0$, $\varepsilon \leqslant \varepsilon_0$. By Assumption 1, there exists $T(\varepsilon) > T_0$, for which

$$\|\Phi(T(\varepsilon)) - E\| < \frac{\varepsilon}{C+1}.$$

We define the controls of the pursuers $P_i, i \in \Lambda_0$ $i \in I$, as follows. If at the moment $t \geqslant t_0$ the number $F_{i1}(t) < 1$, then $u_i(t) \in V$, $m_i(t) \in M_{i1}$ are chosen as the lexicographic minimum among solutions of the equation

$$u_i(t) = v(t) - \lambda(\Phi(t)z^0_{i1}, v(t))\Phi(t)(z^0_{i1} - m_i(t)).$$

If τ_i is the first moment of time for which $F_{i1}(\tau_i) = 1$, then we assume that $\lambda(\Phi(t)z^0_{i1}, v(t)) = 0$ for all $t \geqslant \tau_i$. So $u_i(t) = v(t)$ for all $t \geqslant \tau_i$.

Then from (2), we obtain

$$z_{i1}(T(\varepsilon)) = \Phi(T(\varepsilon))\Big(z^0_{i1} - \int_{t_0}^{T(\varepsilon)} \lambda(\Phi(s)z^0_{i1}, v(s))(z^0_{i1} - m_i(s))ds\Big)$$

$$= \Phi(T(\varepsilon))z^0_{i1}\Big(1 - \int_{t_0}^{T(\varepsilon)} \lambda(\Phi(s)z^0_{i1}, v(s))ds\Big) + \Phi(T(\varepsilon)) \int_{t_0}^{T(\varepsilon)} \lambda(\Phi(s)z^0_{i1}, v(s))m_i(s)ds.$$

By the definition of T_0, $F_{i1}(T_0) = 1$ for some $i \in I$. Consequently, $F_{i1}(T(\varepsilon)) = 1$. Therefore,

$$z_{i1}(T(\varepsilon)) = \Phi(T(\varepsilon)) \int_{t_0}^{T(\varepsilon)} \lambda(\Phi(s)z^0_{i1}, v(s))m_i(s)ds$$

$$= \int_{t_0}^{T(\varepsilon)} \lambda(\Phi(s)z^0_{i1}, v(s))m_i(s)ds + (\Phi(T(\varepsilon)) - E) \int_{t_0}^{T(\varepsilon)} \lambda(\Phi(s)z^0_{i1}, v(s))m_i(s)ds.$$

Note that $\int_{t_0}^{T(\varepsilon)} \lambda(\Phi(s)z^0_{i1}, v(s))m_i(s)ds \in M_{i1}$ and

$$\Big\|(\Phi(T(\varepsilon)) - E) \int_{t_0}^{T(\varepsilon)} \lambda(\Phi(s)z^0_{i1}, v(s))m_i(s)ds\Big\| \leqslant \frac{\varepsilon}{C+1} \cdot C < \varepsilon.$$

Therefore, $z_{i1}(T(\varepsilon)) \in M_{i1} + \mathrm{Int} D_\varepsilon(0)$. The theorem is proved. □

4. Multiple Capture of Given Number of Evaders

Assumption 3. For each $s \in \{0, \dots, q-1\}$, the following is true: for every set $N \subset I$, $|N| = n - sr$, there is a set $M \subset J$, $|M| = q - s$, such that for any $\beta \in M$,

$$\delta_N(\beta) = \min_{v \in V} \max_{\Lambda \in \Omega_N(r)} \min_{\alpha \in \Lambda} \lambda(z^0_{\alpha\beta}, v) > 0$$

Theorem 2. *Suppose that Assumption 1, 3 holds. Then in the game $\Gamma(n, m)$, r-multiple capture of at least q evaders occurs.*

Proof. Let the conditions of the theorem be satisfied. We prove that any $n - sr$ pursuers carries out r-multiple capture of at least $q-s$ evaders, where $s \in \{0, \dots, q-1\}$. Hence, for $s = 0$ we obtain the assertion of the theorem. We prove it by induction. Suppose that $s = q-1$ and $N \subset I$, $|N| = n-(q-1)r$. By the hypothesis of the theorem, there exists $\beta \in J$ such that $\delta_N(\beta) > 0$. Then, by Theorem 1, the pursuers $P_\alpha, \alpha \in N$, carries out r-multiple capture of the evader E_β.

Suppose that we have proved the assertion for all $s \geqslant p+1$. Let us prove the assertion for $s = p$. Let $N \subset I$, $|N| = n - pr$. Then there exists a set $M \subset J$, $|M| = q - p$, such that $\delta_N(\beta) > 0$ for all $\beta \in M$.

Let $v_j, t \in [t_0, \infty), j \in J$, be the set of controls of evaders $E_j, j \in J$. Consider the sets

$$J_\beta = \{\alpha \in N\colon \text{ the pursuer } P_\alpha \text{ catches the evader } E_\beta\}.$$

By Theorem 1 and the conditions of theorem, the inequality $|J_\beta| \geqslant r$ holds for all $\beta \in M$. We can assume that $M = \{1, \dots, q-p\}$. There are two possible cases.

1. $\left|\bigcup\limits_{\beta=1}^{l} J_\beta\right| \geqslant lr$ for all $l = 1, \dots, q-p$. Then by Hall's theorem [15] for sets $\{J_\beta, \beta \in M\}$ there exists a system of different representatives. This means that there are sets J'_β, $\beta \in M$, for which

$$J'_\beta \subset J_\beta,\ |J'_\beta| = r \ \text{ for all } \beta \in M,\ J'_{\beta_1} \cap J'_{\beta_2} = \emptyset \ \text{ for all } \beta_1 \neq \beta_2.$$

Consequently, each group of pursuers $P_\alpha,\ \alpha \in J'_\beta$, carries out r-multiple capture of the evader E_β for all $\beta \in M$. Therefore, the group of pursuers $P_\alpha, \alpha \in N$, carries out r-multiple capture of at least $q-p$ evaders.

2. There exist $l \in \{1, \dots, q-p\}$ such that $\left|\bigcup\limits_{\beta=1}^{l} J_\beta\right| < lr$. Let l be the smallest of the numbers satisfying the given condition. Note that $l > 1$ and $\left|\bigcup\limits_{\beta=1}^{n_1} J_\beta\right| \geqslant n_1 r$ for all $n_1 \in \{1, \dots, l-1\}$. Therefore, for sets J_β, $\beta = 1, \dots, l-1$, there exists a system of different representatives such that

$$J'_\beta \subset J_\beta,\ |J'_\beta| = r \ \text{ for all } \beta = 1, \dots, l-1,\ \ J'_{\beta_1} \cap J'_{\beta_2} = \emptyset \ \text{ for all } \beta_1 \neq \beta_2.$$

Consequently, each group of pursuers $P_\alpha, \alpha \in J'_\beta$, carries out r-multiple capture of the evader E_β. Therefore, the group of pursuers $P_\alpha, \alpha \in \bigcup\limits_{\beta=1}^{l-1} J'_\beta$ carries out r-multiple capture of $l-1$ evaders. In the future, we can assume that the $J'_\beta = J_\beta$ for all $\beta = 1, \dots, l-1$.

Let $s_0 = p + l - 1$. Then, $s_0 > p$ and $s_0 \leqslant q - 1$.

Consider the set $N_1 = N \setminus \bigcup_{\beta=1}^{l-1} J'_\beta$. We have $|N_1| = n - pr - (l-1)r = n - s_0 r$. By hypothesis, there exists a set $M_1 \subset J$, $|M_1| = q - s_0$, such that $\delta_{N_1}(\beta) > 0$ for all $\beta \in M_1$. Note that $\{1, \ldots, l-1\} \cap M_1 = \emptyset$, since if β belongs to a given intersection, then there exists a number $\alpha \in N_1$ for which the pursuer P_α catches the evader E_β, $\beta \in \{1, \ldots, l-1\}$, which would contradict the construction of the set N_1. By virtue of the induction assumption, the group of pursuers P_α, $\alpha \in N_2$, carries out r-multiple capture of at least $q - s_0$ evaders. Consequently, the pursuers P_α, $\alpha \in N$, carries out r-multiple capture of at least $q - s_0 + l - 1 = q - p$ evaders. The theorem is proved. $\square$

Corollary 1. [16] *Suppose that the Assumption 1 holds, V is a strictly convex compact set with a smooth boundary, and for each $s \in \{0, \ldots, q-1\}$ the following is true: for every set $N \subset I$, $|N| = n - sr$, there is a set $M \subset J$, $|M| = q - s$, such that*

$$0 \in \bigcap_{\Lambda \in \Omega_N(n-r+1)} \operatorname{Intco}\{z^0_{\alpha\beta} - M_{\alpha\beta}, \alpha \in \Lambda\} \text{ for all } \beta \in M.$$

then in the game $\Gamma(n, m)$, r-multiple capture of at least q evaders occurs.

Corollary 2. [17] *Suppose that $A(t) = 0$ for all $t \geqslant t_0$, and the Assumption 3 holds; then in the game $\Gamma(n, m)$, r-multiple capture of at least q evaders occurs.*

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