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ON n-VERTEX HAMILTONIAN GRAPHS. I (ORE-TYPE CONDITIONS AND U-GRAPHS)

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Abstract: The Hamiltonian property of n-vertex graphs is studied both in the context of an axiomatic approach and in terms of classical sufficient conditions. It is known that almost all n-vertex graphs of small diameter $k = 1, 2, 3$ are Hamiltonian [11]. It turns out that classical sufficient Ore-type conditions for Hamiltonianity are not well adapted to identifying axiomatically rich classes of such n-vertex graphs and do not explain their Hamiltonian behavior.

Therefore, in this paper, we introduce a new sufficient condition that defines the class of U-graphs based on the metric structure of spheres and the local independence number. We prove that every U-graph is Hamiltonian. Furthermore, almost all n-vertex graphs of fixed diameter $k = 1, 2, 3$ are shown to be U-graphs sharing a unified mechanism for their Hamiltonian property. At the same time, numerous rich, especially exponentially large, classes of U-graphs of diameter 3 and 4 are explicitly constructed outside the known classical Ore-type conditions. Moreover, these well-known classes of Hamiltonian n-vertex graphs are rigorously established to be asymptotically small relative to U-graphs. Thus, for the study of the Hamiltonian property, the discovered condition proves to be more sensitive to the internal structure of graphs, demonstrating wide applicability from both the axiomatic and classical viewpoints.

Keywords: graph, diameter, Hamiltonian graph, Hamiltonian cycle, typical graphs, almost all graphs.

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