Associated editor's report on the paper
"The probability of non-extinction for two branching processes in a joint random environment"
by D.A. Arapov.

The author studies the probability of co-existence of two critical branching processes in a joint random environment. The main theorem shows that this probability decays as n^{-a}, where a depends on the correlation coefficient of associated random walks of the branching processes. The author assumes that this correlation coefficient is positive. The result of the paper is therefore a partial generalisation of [9], where the co-existence probability were studied under the assumption that (random) offspring distributions are geometric.

In my opinion, the generalisation to the case of arbitrary offspring laws is quite important and the paper deserves to be published. But the text should be rewritten almost completely.

Major remarks.

- The work with the existing literature is not careful enough.

  First, the author writes in the abstract "The paper introduces the model..." But this model is already known and has been introduced in a paper by Liu and Vatutin.

  Second, it is a rather strange idea to ignore [9]. The author writes that he 'has discovered [9] during the preparation' of his text. But I think that the time gap of 15 months is too large and that some parts are rather similar. For example, Theorem 5 is just a lemma in [9] and also the proof is very similar. (In both cases, this is an analogue of Lemma 5.2 in [10].) Furthermore, the proof of Lemma 2 is similar to the the corresponding argument in [9].

  Third, The author writes that the results of [9] are proved under superfluous moment restrictions on the associated random walks. But the only difference is the cosmetic assumption on the variances, the assumption on the moments of order p is needed to deal with the case of negative correlation coefficient. This case is much more subtle and needs more accurate treatment.

- The text is too long. Despite of places which are similar to [9], I would suggest to shorten the section on conditioned random walks. Its current version is too detailed for a journal paper.

Minor remarks.

- Page 145, line 13: Here and in some places later, different symbols $\mathbf{f}$ are used.
- Page 158, middle part: One can not extend a harmonic function in a cone to a harmonic function in the whole space. H3 can be valid in the cone only.
- Page, 168, line -2: This statement is valid for $p < 2$ only

Besides my remarks on the existing text, I would like to encourage the author to look at the case of negative correlation coefficients. In my opinion, this case is much more interesting.

I hope that the remarks above will help to produce an improved version.

With best regards,
Vitali Wachtel