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May 17, 2026

The Editor-in-Chief
Siberian Electronic Mathematical Reports

Dear Editor,

We hereby submit our original research manuscript entitled

"On the Inverse Problem of the Exponential Gutman Index of Trees"

by **Jasem Hamoud** and **Duaa Abdullah**, for consideration for publication in *Siberian Electronic Mathematical Reports*.

Background and Motivation. Topological indices are numerical graph invariants computed from the structure of a molecular graph, it plays a central role in mathematical chemistry and combinatorics. Among distance and degree-based indices, the *Gutman index* $\mathrm{Gut}(G) = \sum_{\{u,v\}} d_u\, d_v\, \mathrm{dist}(u,v)$ has attracted substantial research attention since its introduction in 1994, with well-known extremal results on trees characterised by paths and stars. A natural and increasingly active direction is to replace polynomial weightings by exponential ones. In this paper, we study the *exponential Gutman index*

$$\mathrm{HGut}(G) \;=\; \sum_{uv\,\in\,E(G)} \big(2^{\,d_u\,d_v} + dist(u,v)\big),$$

where the sum is taken over the edges of the graph. This index is far more sensitive to high-degree adjacencies than its polynomial counterparts, giving rise to a markedly different extremal theory.

The paper focuses on the class of trees on n vertices and makes the following contributions.

1. **Minimum tree (Theorem 3.3).** We prove by strong induction that the path $\mathcal{P}_n$ is the unique minimiser of HGut among all trees on $n \geqslant 5$ vertices, with $\mathrm{HGut}(\mathcal{P}_n) = 16n - 40$.

2. **Maximum tree (Theorem 3.5).** We prove that the *balanced double star* $DS\big(\lfloor n/2\rfloor, \lceil n/2\rceil\big)$ is the unique maximiser of HGut among all trees on $n \geq 5$ vertices, with

   $$\mathrm{HGut}\big(DS(a,b)\big) = 2^{ab} + (a-1)\cdot 2^a + (b-1)\cdot 2^b, \quad a = \lfloor n/2\rfloor,\ b = \lceil n/2\rceil.$$

   The significant tool is a sharp edge degree-product bound, in any tree on n

vertices, every edge (u, v) satisfies $d_u d_v \leqslant \lfloor n/2 \rfloor \lceil n/2 \rceil$. We emphasise that the star $K_{1,n-1}$ the maximiser for many classical indices is *far from optimal* for HGut, underscoring the fundamentally different extremal structure that the exponential weighting produces.

3. **Complete binary trees (Lemma 3.6).** and **Analytic bounds (Lemmas 3.7–3.8).** We establish a lower bound via Jensen's inequality ($\mathrm{HGut}(G) \geq m \cdot 2^{M_2(G)/m}$, where M_2 is the second Zagreb index) and an upper bound via the Moore bound.

We believe this manuscript is well suited to *Siberian Electronic Mathematical Reports*. The results are complete, all proofs are rigorous, and the exposition is self contained.

Declarations. We confirm that:

- This manuscript is original and has not been published previously, nor is it under consideration by any other journal.
- All authors have read and approved the final version.
- The authors declare no conflicts of interest.
- No external funding was received for this research.
- Data sharing is not applicable; all material is contained within the manuscript.

We would be grateful if you would consider our manuscript for publication in *Siberian Electronic Mathematical Reports*. We welcome the opportunity to revise the work in light of any referee feedback. Please do not hesitate to contact us if you require any further information.

Sincerely,

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