Review of ”Bounds for the success probability in the Odds Theorem associated with a given optimal stopping time“
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The paper under review establishes upper and lower bounds on the success probability in the Bruss Odds Theorem. This theorem solves the following sequential stopping problem. A sequence of independent Bernoulli random variables $X_1, \dots, X_n$ with respective parameters $p_1, \dots, p_n$ is observed at time instances $t = 1, \dots, n$. The goal is to stop ”at the last success“ with maximal probability, i.e., to develop a stopping time τ such that the probability $P(X_\tau = 1, X_{\tau+1} = \cdots = X_n = 0)$ will be maximal. The Bruss Odds Theorem proves the the optimal stopping rule in this problem is to stop the first time s such that the sum of odds $R_s := \sum_{t=s}^n r_t = \sum_{t=s}^n (p_t/q_t)$ is greater than 1; here $q_t = 1 - p_t$. Formally, $s := \max\big\{1, \max\{1 \le t \le n : R_t \ge 1\}\big\}$. The corresponding optimal value is

$$V_n = \sum_{t=s}^n r_t \prod_{t=s}^n q_t = R_s \prod_{t=s}^n q_t.$$

The paper under review derives upper and lower bounds on V_n in terms of R_s, n and s.

Remarks

1. It is unclear what is the motivation for deriving bounds on the optimal value in terms of R_s, n and s when the optimal value V_n is given by a simple explicit formula. The authors write that the success probabilities $p_1, \dots, p_n$ might be unknown, and then the bounds can be useful. However, in this situation it is unclear how the optimal stopping time s can be known. In contrast, Bruss (2003) derives a uniform lower bound on V_n: independently of the problem parameters, $V_n > 1/e$ whenever $R_s \ge 1$.

2. The derivation of upper and lower bounds on the optimal value is much more meaningful and interesting in stopping problems in which there are no explicit formulas for V_n exists.

3. It is quite easy to produce tight bounds on V_n by manipulating the formula for V_n. Such derivations follow elementary calculations and do not require any probabilistic arguments. The inequality between the arithmetic and geometric means immediately leads to the lower bound

$$V_n = R_s \prod_{t=s}^n q_t = \frac{R_s}{\prod_{t=s}^n (1 + r_t)} \ge \frac{R_s}{(1 + \frac{1}{n-s+1} R_s)^{n-s+1}}.$$

This bound is stated as inequality (2.10) in the proof of item 2 of Theorem 3, it holds for all values of R_s, not only for the range $1 \leq R_s \leq 1 + 1/(n-s)$. This bound is tighter than the one given in the item 2 of Theorem 3. Is there any reason to state a weaker result in the theorem? Also, how this lower bound compares to the one stated in part 3? Anyway, the optimal value is known to satisfy $V_n \geq R_s e^{-R_s}$ [see Bruss (2000, 2003)], so the differences between the bounds are insignificant.

4. It seems that the authors are not aware of more recent results on the odds theorem and its variants; see, e.g., the following papers and references therein:

Bruss, T. and Louchard, G. (2009). The odds algorithm based on sequential updating and its performance. *Adv. Appl. Probab.* **41**, 131-153.

Dendievel, R. (2013). New developments of the odds-theorem. *Math. Scientist* **38**, 110-122.

Bruss, T. (2019). Odds–theorem and monotonicity. *Mathematica Applicanda* **47**, 25–43.

Goldenshluger, A., Malinovsky, Ya., and Zeevi, A. (2020). A unified approach for solving sequential selection problems. *Probab. Surveys* **17**, 214-256.