

Response to the Reviewers' Comments

1. *The title and intro talks about dependent observations. This suggests a generality that is not maintained. One should find a more specific title referring to generated by fractional Gaussian noise or Gaussian copula or anything similar.*

Here's the new title:

Number of Distinct Values in a Large Sample with Dependent Observations generated by fractional Gaussian noise from an Infinite Discrete Distribution.

2. *abstract: 'mathematical expectation' is an uncommon word in English.*

I have revised the abstract.

3. *intro: Recall the results from [1] and [2] in the introduction*

Since this article does not deal with the law of large numbers or the central limit theorem, I do not think there is any point in recalling the results from [1,2]. The references to [1,2] are quite sufficient.

4. *Sort the list of references alphabetically*

In this journal, references are numbered as they appear in the text.

5. *The notation $T^{(i)}$ and $T^{(d)}$ are (a) never used and (b) misleading as i and d are typically indices.*

Changed to: Additionally, we compare $\mathbf{ET}_n^{(1/2)}$ and $\mathbf{ET}_n^{(H)}$ ($H \neq 1/2$), where the superscripts denote the independent and dependent cases for the sequence $\{X_k\}$, respectively (a detailed definition is provided below).

6. *Sec 2.1: First sentence. It would be shorter and easier to understand if one only gives the definition without corresponding to... and without the parenthesis.*

Changed to: We denote by F^{-1} the quantile transformation of the function F , defined as $F^{-1}(t) := \inf\{x : F(x) \geq t\}$.

7. *I do not understand at which place in the paper the assumption $H \geq 1/2$ is used. This definitely has to be made clear!*

After Theorem 2, I added the following note: Note that in the proof of this theorem, the condition $H > 1/2$ plays a crucial role, as it ensures the positivity of the covariance function $\rho(\cdot)$ (see the proof of Corollary 1).

8. *The notation $\Phi_{0,1}$ is overly complicated. Φ suffices.*

Corrected.

9. *Remark 1, Reference to [3]. Note that [3] is a book. Give a more concrete reference.*

I clarified the link: Proposition 7.2.10 of [3]

10. *Remark 1, Reference to [5]: This is certainly not the standard reference for LRD.*

I added a link to a well-known book: Beran, J., 1994. Statistics for Long-Memory Processes, Chapman and Hall, New York.

11. *Remark 2 seems like a digression. I'd suggest to either write more so that one can understand the context or delete this.*

I deleted the Remark.

12. *After Theorem 1: Delete From*

Done

13. *After Lem 1: Mention $c < d$.*

Done

14. *Lem 1: This seems like a technical result. Why does it appear in the Theorems section?*

This lemma explains that Theorem 2 is based on a comparison of distributions of a special type. I believe this lemma should be included in this section.

15. I believe that (3) for $c = -\infty$ is Slepian's lemma. This should be cited and referenced correctly.

I made a link to: Slepian, D., 1962. The one sided barrier problem for Gaussian noise, Bell. Syst. Tech. J. 41, 463–501.

16. *In Section 2.3, the same symbol n is used both for the number of Monte Carlo replications and for the sample size.*

In this section, the sample size does indeed equal the number of independent replicates of that sample.

17. *Text below Figure 2: Again the digression on word generation. This might be interesting if explained correctly.*

I deleted this place.

18. *I do not immediately understand (6). It think it is not clearly as mentioned before.*

I clarified that point: Given that $a_n \rightarrow +\infty$ and $b_n \rightarrow +\infty$ as $n \rightarrow +\infty$ (where a_n, b_n are defined in Lemma 2), we deduce that for all sufficiently large n , the following inequality holds ...

19. *There are results similar to Proposition 1 in the paper: Wenbo V. Li and Qi-Man Shao, A normal comparison inequality and its applications, Probability Theory and Related Fields 122 (2002), 494–508. There are other papers referenced by that paper and other papers that reference that paper. I think this more modern literature should be included. And it should be checked if that helps to extend the results.*

I've added a link to this paper. I'll be looking into related works in the future. Thanks!

20. *The notation in Prop. 1 is inconsistent:*

Corrected

21. *In (12) to (13), the application of Jensen is quite quick. Insert a step.*

Done.

22. *Lemma 3 is just Slepian's lemma. Reference correctly.*

Done.

23. *I cannot follow the proof of Lemma 1. The word 'immediately' is certainly inappropriate.*

I have provided a detailed proof.

24. *The list of references looks inconsistent. In [6] it says "In-formational"?*

Corrected.