Report on the paper

"On the accuracy of approximations for Wiener probabilities to stay between square root boundaries"

by A.I. Sakhanenko

Let $B(t)$ be a standard Brownian motion. For any pair of constants $-\infty < c_1 < c_2 \le \infty$ we define

$$\tau_{c_1,c_2} := \inf\{t > 1 : B(t) \notin (c_1\sqrt{t}, c_2\sqrt{t})\}.$$

The study of the tail behaviour of such stopping times and of corresponding conditional distributons is one of the natural tasks in the theory of stochastic processes. Breiman and Uchiyama have derived main terms in the asymptotic behaviour of the mentioned above probabilities. The next step is to understand the quality of such approximations, which control all the parameters of the problem in a more or less explicit way. Such an information is crucial when one studies first passage times for $B(t)$ when the boundaries only asymptotically proportional to $c_i\sqrt{t}$ or, what is more important, when one studies first-passage times for random walks in discrete time.

The author proves such estimates, which generalize the results of Uchiyama who has considered the case $c_2 = \infty$. I have checked all the proofs and believe that they are correct. So, i have no hesitation in recommending this paper for publication in the *Siberian Electronic Mathematical Reports.*

Remarks.

p.1: I would reformulate the last sentence in the abstract. How can you improve estimates obtained for the problem with one boundary?

p.1, l.-1: I would write $\inf\{t > 1 \ldots\}$, since if $B(0) = 0$ then $\inf\{t > 0 : U_t \notin (c_1, c_2)\} = 0$ for all finite constants c_i. A further reason for this change is your equation (1), where only times larger than 1 are considered.

p.2, l.-13: I would write 'for all $c_1 < c_2$' instead of 'in domain $\mathcal{D}$', since all other parameters are not used here.

p.2, l.-2: Split this display into two parts.

p.3, l.12: I would first mention that $\kappa(c) > 1/2$ for all c. Your current sentence can be understood as follows: $\kappa(c)$ is continuous on the set $\{c : \kappa(c) > 1/2\}$.

p.3, l.16: What is u_a here? I have not found the definition of this function. If it is a free parameter, then I would like to see a discussion, why it is not sufficient to take $u_a = 2$.

p.3, l.19: Remove two obvious inequalities in this line.

p.3, l.20: 'partial' s/b 'particular'

p.4, l.3: Shift this sentence to Remark 1. It is hard to undestand this sentence before (12) and (15) have been read.

p.4, l.-14: Remove the comma after 'better'.

p.4, l.-7: 'satisfy the next properties' sounds strange.

p.4, l.-3: 'Theorem 2' s/b 'Theorem 1'.

p.5, l.7: The statements of Theorems 2 and 3 look very different. Why does it make sence to mention that (17) is less restrictive than (20). Anyway, this is quite obvious.

p.5, l.-12: Could you give a reference for (24)? I think that u_0 should be ω_0 in this formula.

p.5, l.-1: 'Uhlenbeck' s/b 'Ornstein–Uhlenbeck'

p.6, l.-7: I would mention the connection between eigenvalues λ_i and the numbers λ and κ from your theorems.

p.6, l.-4: I would write 'where the functions ϕ_k and the numbers λ_k are defined in (25)'

p.7, l.3: v_x, u and v shlud be v_y, u_a and v_y respectively.

p.7, l.12: Remove | on the left hand side of this formula.

p.7, l.12: I would write 'consequence' instead of 'corollary'.

p.8, l.-16: 'domain (2)' should be 'domain $\mathcal{D}$'; 'takes' should be 'take'

p.8, l.-15: ψ should be ϕ in Lemma 4

p.8, l.-7: I do not think that one should mention the inequality $2 > 1$

p.9, l.1: ψ should be ϕ

p.9, l.-9: Fullstop is missing in this line. I would also add comma after (35) p.9, l.-7: I think it should be 'arrive at'

p.9, l.-1: Please change the variable in the integral, y is the upper limit of the interval. This should also be done in (43)

p.10, l.8: Remove the word 'special'. Typically, the phrase 'special functions' refer to a concrete class of functions (Bessel, Whittacker and so on)

p.10, l.-2 I would remove the word 'next'