

**REFEREE'S REPORT ON THE ARTICLE  
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WEAK SOLVABILITY OF ONE ALPHA-MODEL FOR  
VOIGT-TYPE FLUID WITH INFINITE MEMORY,  
SUBMITTED TO  
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The paper deals with the boundary value problem for the mathematical model motion of viscoelastic fluids with memory. In considered model it is assumed that the behavior of the material is governed by the fractional Voigt-type constitutive law. The novelty of the work is that the authors consider the explicit constitutive law with the infinite memory term concentrated on the trajectories of material particles. Since the velocity field is not sufficient smooth to uniquely determine the trajectories, the authors using regular Lagrangian flows. This makes it possible to formulate the problem correctly. For the formulated viscoelastic fluid with memory, the alpha model is studied in this paper.

The idea of using such models was first suggested by J. Leray in order to prove the existence of a weak solution of the Navier–Stokes system of equations. Each alpha model is characterized by its own vector-valued first-order differential operator  $F(u, v)$  whose components are linear combinations of all possible operators of the form  $u^k \partial_{x_j} v^n$ ,  $v^k \partial_{x_j} u^n$ ,  $u^k \partial_{x_j} u^n$ . Alpha models are divided into two classes, depending on the orthogonality properties of the function  $F(u, v)$  to  $u$  or  $v$  in the space  $H$ . I class:  $\langle F(u, v), v \rangle = 0$ ; II class:  $\langle F(u, v), u \rangle = 0$ . In this paper the authors consider the I class alpha model for the formulated viscoelastic fluids with memory.

The main result is the proof of the existence of global weak solution to this problem in 2D and 3D case. Moreover, the authors establish a convergence of solutions of alpha model to the solution of classical model as the parameter alpha tends to zero. The main research method in the work is topological approximation method. For this, a two parameter family of auxiliary problems depending on  $0 < \xi < 1$  and  $\varepsilon \leq 0$  on the time interval  $[-m, T]$  is introduced. For this family a priori estimates are obtained. Using topological degree theory for condensing vector fields, the existence of weak solutions for auxiliary problem with  $\xi = 1$  is proved. Then, the authors receive the additional necessary estimates for solutions of the auxiliary problem with  $\xi = 1$  and obtain the passage to the limit for  $m \rightarrow \infty$  and  $\varepsilon \rightarrow 0$ . Finally, the authors show that the sequence of solutions of alpha models converges to solution of the classical problem.

The results presented are nontrivial, original, and interesting. The paper is well written and structured, the explanation is clear. The work is of interest to specialists in partial differential equations, fluid dynamics, nonlinear functional analysis.

I recommend the paper “Weak solvability of one alpha-model for Voigt-type fluid with infinite memory” by A.V. Zvyagin and E.I. Kostenko for publication in Siberian Electronic Mathematical Reports.