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NON-LINEAR TEMPORAL LOGIC, ADMISSIBILITY FOR RULES AND ALMOST-PROJECTIVE FORMULAS

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Abstract: In this paper we study admissibility problem for non-linear temporal logic L. We consider a special generalization of projective formulas. Using this technique we find algorithm computing most general unifier for any given unifiable in L formula. Logic L is generated by family of all closed temporal models with compression property. Based on prepared technique, we prove that the admissibility problem and unification problem for L are decidable.

Keywords: temporal logic, unification, admissibility problem, computation of unifiers, projective formulas, admissible rules

1 Introduction, background

In logic and computer science, specifically automated reasoning, unification is an algorithmic process of solving equations between symbolic expressions. This technique often used in automated deduction, optimization programs and pure mathematical logic (cf. Robinson [17], Knuth et al [16],

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Baader and Snyder [1], Baader and Ghilardi [2]). In particular it is a prominent instrument for verification admissibility for inference rules. For example, Harvey Friedman problem about recognoscing admissibility for inference rules in the intuitionistic propositional logic INT has close connection with properties of unification (the Friedman problem was solved by V.Rybakov (in 1984, cf. for reference and history the book [20]). And later in 1997 Silvio Ghilardi [7, 8] found another solution of admissibility problem by usage projective formulas and unification (cf. S. Ghilardi [7, 8, 9]). This view on admissible rules was developed longer by several authors (cf. eg. Jerábek [14, 15], Iemhoff, Metcalfe [12, 13], Balbiani at all [3, 4], Bashmakov at all [5]). W.r.t. admissibility, a generalization of this problem on inference rules which may contains parameters (looks the same as coefficients in algebraic equations) is more interesting and more complicated. This problem was solved by Rybakov [18, 19, 20] for intuitionistic logic and modal logics $S4$ and Grz.

We will work in this paper with a variation of temporal logic and unification. As well known in mathematical logic and philosophy, temporal logic is a mathematical symbolism for representing, and reasoning about events qualified in terms of time. In literature, temporal logic sometimes used to refer to tense logic, as – a modal logic-based system of temporal logic which was introduced by Arthur Prior in the late 1950s. It has been further developed and applied in computer scientists, notably by Amir Pnueli, and other logicians (cf. e.g. Gabbay at al [10, 11]), cf. [21, 23, 24]).

Linear modal logic may be seen as linear temporal logic (logic with linear time). Studying it Dzik and Wojtylak [6] proved that any formula unifiable in the linear modal logic $S4.3$ is projective, which gives direct solution of admissibility problem for $S4.3$. (Ideas similar to projectivity for linear modal and intuitionistic logics were suggested already in A. Wroński [25, 26]).

At the same time, progress for admissibility problem in temporal logic itself was rather not so successful, many problems remain open. Even until now the admissibility problem was not solved for none-linear temporal logic with transitive (accessibility by time) relation. In this paper we consider admissibility problem in some such non-linear temporal logic. We assume that L is generated by family of all possible temporal models which are closed and have the compression property. We consider a modification of projective formulas and by it we find algorithm computing most general unifier for any given unifiable in L formula. So we prove that unifiability problem and admissibility problems for such logic L are decidable.

2 A very few known definitions and denotation

We start from definition the syntax for our temporal logic. It contains the set $Prop$ of propositional letters and Boolean logical operations.

As for additional temporal logical operations we have a choice for using and fixing notation. First, paying respect to Prior for his invention, we may consider only Prior's choice for operations: unary logical operation G (means – it always will be true) and unary logical operation H (it always was true in past). So for a formula φ, $G\varphi$ is meant as: always in future φ will be always true, $H\varphi$ says that always in past φ was true.

Second way, besides we may use only modified standard modal logical operations $\Box^+$ (for future) and $\Box^-$ for past and they may be expressed by standard mentioned temporal ones: $\Box^+\varphi := G\varphi$, $\Box^-\varphi := H\varphi$. And wise versa $G\varphi := \Box^+\varphi$, $H\varphi = \Box^-\varphi$. Later we will prefer to use only $\Box^+$ and $\Box^-$ as additional temporal operations.

Now we briefly recall (well known) definitions for semantic of modal and temporal bi-models. Standard semantics for modal propositional logics consists of frames $F := \langle W, R\rangle$ (that are sets W of possible states (worlds) with a binary accessibility relation R on W; so, if $a, b \in W$ and aRb we say that b is accessible from a). Models are obtained from the frames by introduction valuations V for some chosen sets of propositional letters $Prop$. So, for any $p \in Prop$, $V(p) \subseteq W$, $V(p)$ is the set of all w from W where p is true (w.r.t. V). The triple $M := \langle W, R, V\rangle$ is said to be a Kripke model.

For any Kripke model M, the truth values can be extended from propositions of $Prop$ to arbitrary (bi-modal) formulas as follows:

$$\forall p \in Prop\ (M, a) \Vdash_V p \Leftrightarrow a \in W \wedge\ a \in V(p);$$

$$(M, a) \Vdash_V (\varphi \wedge \psi) \Leftrightarrow (M, a) \Vdash_V \varphi\ \wedge\ (M, a) \Vdash_V \psi;$$

$$(M, a) \Vdash_V (\varphi \vee \psi) \Leftrightarrow (M, a) \Vdash_V \varphi\ \vee\ (M, a) \Vdash_V \psi;$$

$$(M, a) \Vdash_V \neg\varphi \Leftrightarrow not[(M, a) \Vdash_V \varphi];$$

$$(M, a) \Vdash_V \Box^+\varphi \Leftrightarrow \forall b[(a\ R\ b) \Rightarrow (M, b) \Vdash_V \varphi];$$

$$(M, a) \Vdash_V \Box^-\varphi \Leftrightarrow \forall b[(b\ R\ a) \Rightarrow (M, b) \Vdash_V \varphi].$$

For a Kripke model $M := \langle W, R, V\rangle$ and a formula φ with letters from the domain of V, φ is valid in M (denotation – $M \Vdash \varphi$) if, for any b of W, the formula φ is true at b (denotation: $(M, b) \Vdash_V \varphi$).

Definition 1. For a given class of frames K, the temporal (bi-modal) logic generated by K is the set of all temporal formulas which are true at any state of any model obtained from any frame from K by introduction of any possible valuation of propositional letters; notation $L = L(K)$.

Any modal logic is simply a particular case of bi-modal logic when we use only one modal operation $\Box$ (as $\Box^+$) instead two ones: $\Box^+$ and $\Box^-$.

In this paper we consider only frames with reflexive and transitive accessibility relations R. Logic $L(K)$ itself is decidable if for any formula we may compute if $\varphi \in L(K)$. A formula φ is *satisfiable* in $L(K)$ if there is a model M constructed on a frame from K, where φ is true at some its state.

3 A modification for definition of projective formulas

First we briefly recall the known standard definitions in order to show how our extension works towards standard projective formulas. Our comments below are exactly the same for any modal or bi-modal logic. Let For be the set of all formulas, let P be a set of letters. A mapping ε of P into For is said to be a substitution for P. That substitution ε can be extended to the set of all formulas in letters from P by $\varepsilon(\varphi(x_1,\ldots,x_n)) := \varphi(\varepsilon(x_1),\ldots,\varepsilon(x_n))$. That is similar for all logics.

Definition 2. *A formula φ is unifiable in a logic L if there is a substitution ε which is called a unifier for φ such that $\varepsilon(\varphi) \in L$.*

That is $\varepsilon(\varphi)$ is the result of substitution formulas $\varepsilon(x_i)$ in the formula φ instead any propositional letter x_i occurring in φ. *A unifier ε (for a formula φ in a logic L) is more general than a unifier ε_1 iff there is a substitution δ such that for any letter x, $[\varepsilon_1(x) \equiv \delta(\varepsilon(x))] \in L$.* A set of unifiers CU for a given formula φ in a logic L is a complete set of unifiers, if the following holds. For any unifier σ for φ in L, there is a unifier σ_1 from CU, where σ_1 is more general than σ.

If a logic L is decidable, usually to check the unifiability a formula in L is (theoretically, not computationally) an easy task: it is sufficient to use only ground substitutions: mappings of propositional letters in the set $\{\bot,\top\}$. But the problem - how to find all unifiers - all solving substitutions - is not easy at all. Now we recall the standard definition of projective formulas for modal logics.

Definition 3. *A formula φ is said to be projective in a logic L if the following holds. There is a substitution σ (which is called projective substitution) which is an unifier for φ such that $\Box\varphi \to [x_i \equiv \sigma(x_i)] \in L$ for any letter x_i from φ.*

The use of projective formulas for modal logics comes from the following statement.

Lemma 1. *If a substitution σ_p is projective for a formula φ in a modal logic L, then $\{\sigma_p\}$ is a complete set of unifiers for φ (i.e. σ_p is most general unifier).*

Proof. Indeed, let σ be a unifier for φ in L. Since we assume σ_p is projective for φ in L, we have $\Box\varphi \to [x_i \equiv \sigma_p(x_i)] \in L$ for any letter x_i from φ. Acting by σ on the formula above we get $\sigma(\Box\varphi) \to [\sigma(x_i) \equiv \sigma(\sigma_p(x_i))] \in L$, that is $\sigma(x_i) \equiv \sigma(\sigma_p(x_i)) \in L$. Q.E.D.

It works very well for linear modal logics for finding most general unifiers, but for not-linear ones and moreover for temporal logics that approach cannot be applied directly . Also recall that not all unifiable formulas are projective. Recall (without a proof) a known result concerning the linear temporal logic L (on the set of all natural numbers as the generating frame).

Example (cf. eg. [22]). *Formula* $\varphi = \Box(\Box x \vee (\neg x \wedge N\Box x))$ *is unifiable in* L *but not projective.*

In this our paper we study an non-linear temporal logics with aim to sole problem of computability for admissible rules and for finding most general unifiers. For this we introduce some restriction on temporal logics under our consideration.

Note that we do not consider in our this paper temporal logics with operation *until* (or *since*) because the logic is not linear and so there is no way to define correctly the rules for computation *until* (or *since*). As noted already, a temporal frame is a pair $\langle W, R\rangle$, where W is a set (temporal states) and R is a binary relation on W (accessibility for states, aRb means that the state b is accessible form state a).

We fix that since now, by definition, the relations R are always reflexive and transitive.

To simplify writings we write aR^+b for aRb (with meaning b is future state for a) and aR^-b for bRa (with meaning b is past state for a).

Now we turn to our modification of definition for projective formulas. We need

Definition 4. *We say that a temporal frame* $\langle W, R\rangle$ *is closed if for any two states* $a, b \in W$ *there are states* $a_1, b_1, a_2, b_2, \ \dots \ a_i, b_i, \ \dots \ a_k, b_k \in W$ *such that* $a = a_1,\ b = b_k,$ *and*

$$a_1R^+b_1, b_1R^-a_2, a_2R^+b_2, \ \dots\ , a_iR^+b_i, b_iR^-a_{i+1}, \ \dots\ , a_kR^+b_k.$$

In this case we will say that there is an zig-zag path in W from a to b of length k.

Definition 5. *Let* $k \in N$. *We say a temporal logic* L *has* k*-zig-zag compression if* L *is generated by a family Kripke frames* $\langle W, R\rangle$, *such that for any such* $\langle W, R\rangle$ *the following holds.*

If for any $a_1, b_1, a_2, b_2, \ \dots \ a_i, b_i, \ \dots \ a_n, b_n$ *from* W *with* $n \geq k+1$ *and*

$$a_1R^+b_1, b_1R^-a_2, a_2R^+b_2, \ \dots\ , a_iR^+b_i, b_iR^-a_{i+1}, \ \dots\ , a_nR^+b_n.$$

there are $c_1, d_1, \ \dots \ c_i, d_i, \ \dots \ c_m, d_m \in W$ *such that*

$$c_1R^+d_1, d_1R^-c_2, c_2R^+d_2, \ \dots\ , c_iR^+d_i, d_iR^-c_{i+1} \ \dots \ c_mR^+d_m,$$

and $a_1 = c_1, b_n = d_m,$ *where* $m < k+1$.

That is, if there is a zig-zag path from a_1 to b_n inside W of, say, – zig-zag length n, which is bigger as k, then there is a zig-zag path from a_1 to b_n inside W of length at most k. Recall now definition of main object of our research - admissibility of inference rules.

Definition 6. *Let* $\varphi_1, \dots, \varphi_n, \psi$ *be some formulas. Inference rule compound from this formulas is the expression* $\varphi_1, \dots \varphi_n/\psi$. *It said to be admissible in a logic* L *iff for any substitution* ϵ *if* $\epsilon(\varphi_1) \in L, \dots, \epsilon(\varphi_n) \in L$ *then* $\epsilon(\psi) \in L$.

4 Results on Decidability

We consider now below only non-linear temporal logic L which is generated by all closed temporal frames with k-zig-zag compression (that is L = L(K)). It is clear that a formula φ is unifiable in L iff φ is true in one element reflexive frame. We turn now to find a complete set of unifiers.

For a formula φ and $m \in N$, $[\Box^+\Box^-]^m\varphi$ denotes the formula $(\Box^+\Box^-)(\Box^+\Box^-)...(\Box^+\Box^-)(\Box^+\Box^-)\varphi$ where $(\Box^+\Box^-)$ is taken m times.

Theorem 1. *For (non-linear) temporal logic L generated by all closed temporal frames with k-zig-zag compression there is an algorithm computing most general unifier for unifiable formulas.*

Proof. Given a formula $\varphi(x_1,...,x_n)$. First we verify if it is unifiable in L at all. For this we verify its truth in the one element frame. If for some valuation δ_1, where $\delta_1(x_i) := g_i$ $(g_i \in \{\top, \bot\})$, for its letters x_i, $\delta_1(\varphi) = \top$ holds then the formula $\varphi(x_1,...,x_n)$ is unifiable in L. If for all possible g_i it is not the case then formula φ is not unifiable in L. We assume that $\varphi(x_1,...,x_n)$ is unifiable in L, fix g_i and continue. So, we take and fix a substitution $\sigma_1(x_i) := g_i$ where $\varphi(\sigma_1(x_i),\ldots,\sigma_1(x_n)) \in L$, if no such g_i, we chose and fix g_i to be those which make false φ at the one-element model after substitution g_i in place of x_i.

Now for any letter x_i occurring in φ we define the following substitution:

$$\sigma(x_i) := \Big([\Box^+\Box^-]^{k+1}\varphi(x_1,...,x_n) \wedge x_i\Big) \vee [\neg[(\Box^+\Box^-)^{k+1}\varphi(x_1,...,x_n)] \wedge g_i].$$

Lemma 2. *$\sigma(x_i)$ is a unifier for φ.*

Proof. By our assumption the logic L is generated by a set G of closed models M_m and these models has k-zig-zag compression property. Take a model M_m of this sort. Take and fix a state x from M_m. Only two cases are possible:

(I) $(M_m, x) \nVdash_V [\Box^+\Box^-]^{k+1}\varphi(x_1,...,x_n)$, or

(II) $(M_m, x) \Vdash_V [\Box^+\Box^-]^{k+1}\varphi(x_1,...,x_n)$.

Now it remains to show that the formula $\sigma(\varphi)$ is true at each x from M_m.

Lemma 3. *If (II) it a case then $\sigma(\varphi)$ is true at all states of M_m.*

Indeed, consider any state y from M_m. We know that x is a state of M_m, and since the model M_m is closed, there is zig-zag path P from x to y. Because by condition of this theorem, the frame of the model M_m has k-zig-zag compression, there is a path P from x to y with length at most k.

Then by

$$(M_m, x) \Vdash_V [\Box^+\Box^-]^{k+1}\varphi(x_1,...,x_n)$$

we obtain $(M_m, y) \Vdash_V \varphi(x_1,...,x_n)$. Because this holds for any y from M_m, We get

$$(M_m, y) \Vdash_V [\Box^+\Box^-]^{k+1}\varphi(x_1,...,x_n).$$

Therefore $(M_m, y) \Vdash_V \sigma(x_i)$ for all y from M_m. That is because the truth value of any x_i at any y w.r.t. the original valuation V is exactly the same as the truth value of $\sigma(x_i)$ w.r.t. V. Because $(M_m, y) \Vdash_V \varphi(x_1, ..., x_n)$ we get $(M_m, y) \Vdash_V \sigma(\varphi(x_1, ..., x_n))$. It concludes the proof of this lemma.

Lemma 4. *If (I) holds then $\sigma(\varphi)$ is true at all states of M_m.*

Let

$$(M_m, x) \Vdash_V \neg[\Box^+\Box^-]^{k+1}\varphi(x_1, ..., x_n).$$

Then there is a zig-zag path from x to some y from M_m where

$$(M_m, y) \Vdash_V \neg\varphi(x_1, ..., x_n).$$

Since the model is closed, we have that for any z from M_m there is a zig-zag path from z to y. By k-zig-zag compression property, there is a zig-zag path from z to y of length at most k. So at any z

$$(M_m, z) \Vdash_V \neg[\Box^+\Box^-]^{k+1}\varphi(x_1, ..., x_n).$$

So we get that for any z from M_m, the truth value of any $\sigma(x_i)$ at z is g_i. By choice of g_i we get $(M_m, z) \Vdash_V \sigma(\varphi(x_1, ..., x_n))$. Lemma is proved.

Definition 7. *A set of unifiers CU for a given formula φ in a logic L is a complete set of unifiers, if the following holds. For any unifier σ_1 for φ in L, there is a unifier σ from CU, where σ is more general than σ_1, that is there is a substitution σ_2 such that for any letter x_i, $\sigma_1(x_i) = \sigma_2(\sigma(x_i))$.*

Definition 8. *We tell that a formula φ is almost-projective (denotation – alpr) in our logic L if the following holds. There is a substitution σ (we call it alpr-substitution) which is an unifier for φ and*

$$[\Box^+\Box^-]^{k+1}\varphi \to [x_i \equiv \sigma(x_i)] \in L$$

for any letter x_i from φ.

We see that our definition here is depending on the number k, which is fixed during definition of our logic.

Lemma 5. *If a substitution σ_p is alpr for a formula φ in our logic L, then $\{\sigma_p\}$ is a complete set of unifiers for φ (i.e. σ_p is most general unifier).*

Indeed, Since we assume that σ_p is alpr for φ in L, we have $[\Box^+\Box^-]^{k+1}\varphi \to [x_i \equiv \sigma_p(x_i)] \in L$ for any letter x_i from φ. Take an arbitrary unifier σ_2 for φ. Acting by σ_2 on the formula above we get

$$\sigma_2([\Box^+\Box^-]^{k+1}\varphi) \to [\sigma_2(x_i) \equiv \sigma_2(\sigma_p(x_i))] \in L.$$

Since σ_2 is a unifier for φ, we have $\sigma_2(\varphi) \in L$ and $[\Box^+\Box^-]^{k+1}\sigma_2(\varphi) \in L$. Therefore $\sigma_2(x_i) \equiv \sigma_2(\sigma_p(x_i)) \in L$. Q.E.D.

Lemma 6. *$\sigma(x_i)$ is an alpr for φ.*

Proof. We shown already that σ is a unifier for φ, and it remains to show that

$$[\Box^+\Box^-]^{k+1}\varphi \to [x_i \equiv \sigma(x_i)] \in L$$

Take a closed model M_m with k-zig-zag compression. Take any state y from M_m. σ is, as we proved above, a unifier for φ and hence $\sigma(\varphi)$ is true in any state of M_m. Therefore $(M_m, y) \Vdash_V \sigma(\varphi)$ for any y and for any y, $(M_m, y) \Vdash_V [\Box^+\Box^-]^{k+1}\sigma(\varphi)$. Hence by definition of $\sigma(x_i)$ above the truth value of any x_i at any y w.r.t. the original valuation V is exactly the same as the truth value of $\sigma(x_i)$ w.r.t. V. Q.E.D.

Theorem 2. *Admissibility problem for logic L is decidable.*

Proof. Indeed, for any rule $\varphi_1, \dots \varphi_n/\psi$ we may compute if it is admissible in L as follows. An inference rule $\varphi_1, \dots \varphi_n/\psi$ is not admissible in L iff there is a unifier σ_1 for $\varphi_1 \wedge \dots \wedge \varphi_n$ which is not a unifier for ψ. But if so, then the computed by us above unifier σ_p for $\varphi_1, \dots \varphi_n$ (cf. Lemmas 2, 5, 6) does the same for $\varphi_1, \dots \varphi_n$ and ψ as σ_1 above. $Q.E.D.$

References

[1] F. Baader, W. Snyder, *Unification theory*, In: Handbook of Automated Reasoning. I. Elsevier Science Publishers (2001), 447–533. Zbl 1011.68126

[2] F. Baader, S. Ghilardi, *Unification in modal and description logics*, Log. J. IGPL, **19**:6 (2011), 705–730.

[3] Ph. Balbiani, M. Mojtahedi, *Unification with parameters in the implication fragment of classical propositional logic*, Log. J. IGPL, **30**:3 (2022), 454–464. Zbl 1494.03025

[4] Ph. Balbiani, *Unification in modal logic*, In: 8th Indian conference, ICLA 2019, Delhi, India, March 1–5, 2019, Springer, Berlin, 2019. Zbl 1533.03013

[5] S. Bashmakov, A. Kosheleva, V. Rybakov, *Non-unifiability in linear temporal logic of knowledge with multi-agent relations*, Sib. Èlectron. Mat. Izv., **13** (2016), 656-663. Zbl 1390.03020

[6] W. Dzik, P. Wojtylak, *Projective unification in modal logic*, Log. J. IGPL, **20**:1 (2012), 121–153. Zbl 1260.03041

[7] S. Ghilardi, *Unification Through Projectivity*, J. Log. Comput., **7**:6 (1997), 733–752. Zbl 0894.08004

[8] S. Ghilardi, *Unification in Intuitionistic logic*, J. Symb. Log., **64**:2 (1999), 859–880. Zbl 0930.03009

[9] S. Ghilardi, *Best solving modal equations*, Ann. Pure Appl. Logic, **102**:3 (2000), 183–198. Zbl 0949.03010

[10] D.M. Gabbay, I.M. Hodkinson, M.A. Reynolds, *Temporal logic. Vol. 1. Mathematical foundations and computational aspects*, Clarendon Press, Oxford, 1994. Zbl 0921.03023

[11] D.M. Gabbay, I.M. Hodkinson, *An axiomatization of the temporal logic with Until and Since over the real numbers*, J. Log. Comput., **1**:2 (1990), 229–260. Zbl 0744.03018

[12] R. Iemhoff, *On the admissible rules of intuitionistic propositional logic*, J. Symb. Log., **66**:1 (2001), 281–294. Zbl 0986.03013

[13] R. Iemhoff, G. Metcalfe, *Proof theory for admissible rules*, Ann. Pure Appl. Logic, **159**:1-2 (2009), 171–186. Zbl 1174.03024

[14] E. Jerábek, *Admissible rules of modal logics*, J. Log. Comput., **15**:4 (2005), 411–431. Zbl 1077.03011
[15] E. Jerábek, *Independent bases of admissible rules*, Log. J. IGPL, **16** (2008), 249–267. Zbl 1146.03008
[16] D. Knuth, J. Bendix, *Simple Word Problems in Universal Algebras*, Comput. Probl. abstract Algebra, Proc. Conf. Oxford (1970), 263–297. Zbl 0188.04902
[17] A. Robinson, *A machine oriented logic based on the resolution principle*, J. Assoc. Comput. Mach., **12**:1 (1965), 23–41. Zbl 0139.12303
[18] V.V. Rybakov, *Problems of Substitution and Admissibility in the Modal System Grz and in Intuitionistic Propositional Calculus*, Ann. Pure Appl. Logic, **50**:1, (1990), 71–106. Zbl 0709.03009
[19] V.V. Rybakov, *Rules of Inference with Parameters for Intuitionistic logic*, J. Symb. Log., **57**:3 (1992), 912–923. Zbl 0788.03007
[20] V.V. Rybakov, *Admissible Logical Inference Rules*, Stud. Logic Found. Math., **136**, Elsevier, Amsterdam, 1997. Zbl 0872.03002
[21] V.V. Rybakov, *Linear temporal logic with until and next, logical consecutions*, Ann. Pure Appl. Logic, **155**:1 (2008), 32–45. Zbl 1147.03008
[22] S. Babenyshev, V. Rybakov, *Linear temporal logic LTL: basis for admissible rules*, J. Log. Comput., **21**:2 (2011), 157–177. Zbl 1233.03026
[23] V.V. Rybakov, *Logical consecutions in discrete linear temporal logic*, J. Symb. Log., **70**:4 (2005), 1137–1149. Zbl 1110.03010
[24] V.V. Rybakov, *Writing out Unifiers in Linear Temporal Logic*, J. Log. Comput., **22**:5 (2012), 1199–1206.
[25] A. Wrónski, *Transparent Unification Problem*, Rep. Math. Logic, **29** (1995), 105–107. Zbl 0865.08002
[26] A. Wrónski, *Transparent Verifiers in Intermediate Logics*, Abstracts of the 54-th Conference in History of Mathematics, Cracow 2008.

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