

Review

on the work “Classic solution of the Cauchy problem for a mildly quasilinear wave equation with a variable coefficient” by V.I. Korzyuk and J.V. Rudzko

This work studies the well-posedness of a classic Cauchy problem for one-dimensional wave equation (speed of sound $a(x)$ depends on spacial variable) where the force density, i.e. function f depends on the unknown function u and it's first variables. It is also important that along with the usual smoothness conditions on $a(x)$ and $f(t, x, u(t, x), u_t(t, x), u_x(t, x))$, on initial data $\varphi(x)$ and $\psi(x)$ additionally function f satisfies Lipshitz condition with respect to u, u_t, u_x . This is a principal moment.

This work is written in a clear language main statements and theorems 1,2,3 are well justified. The base idea behind them is to transform to the integral Volterra-type equation then to regularize it and then to use the theorem about the continuous continuation by parameter.

Some unimportant technical remarks.

1. Starting from page 7: instead of $J[u_1](t, x) - J[u_1](t, x)$ should be $J[u_1](t, x) - J[u_2](t, x)$.

2. Inaccurate from of the second inequality in the chain of inequalities (26).

3. Instead of reference [19 p.39, Lemma 1] its better to write something like: using the known Gronwall's lemma...

4. In addition to reference [22] it is also appropriate to reference classic monograph by Gilbarg and Trudinger “Elliptic Partial Differential Equations of Second Order”, Springer, 2001, which should ease reading this work (in reference to the method of continuation by parameter).

Despite its methodical advantages this work in my opinion unfortunately does not contain a new significant scientific result.

I recommend to decline this paper.

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