

# Review of the paper

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## *Surjective Isometry Reflexive Strongly Facially Symmetric Spaces*

The submitted manuscript is devoted to the study of surjective isometries in reflexive strongly facially symmetric spaces (SFS-spaces) and presents a criterion for isometric equivalence formulated via the mapping of the set of indecomposable geometric tripotents. The topic is relevant to specialists in the geometry of Banach spaces and the theory of JB\*-triples. The work is rigorously written, based on modern developments in the field, and contributes to the structural understanding of linear isometries in Banach and triple systems.

However, in order to improve the clarity of exposition and ensure compliance with academic standards, the paper requires several revisions both technical and content-related. Specific remarks and suggestions are provided below.

### Comments and Suggestions

- **Page 144, paper title:** The title contains a grammatical error. It should be corrected to:  
*SURJECTIVE ISOMETRY IN REFLEXIVE STRONGLY FACIALLY SYMMETRIC SPACES*
- **Page 145, line 2 from the top:**  
The sentence **These spaces are considered as a geometric model for representing states in quantum mechanics.**  
is recommended to be rephrased more concisely as:  
**These spaces serve as a geometric model representing states in quantum mechanics.**
- **Page 147, fifth line from the bottom:**  
" $u \diamond v$ .Then" should be replaced with " $u \diamond v$ , then" for correct sentence structure.
- **Page 149, lines 3-4 from the bottom:**  
**an surjective** should be corrected to **a surjective**.
- **Page 149, last line:**  
**iz uniquely determined** should be corrected to **is uniquely determined**.

- **Abstract (Page 144):**

The current abstract is too brief and does not reflect all the key aspects of the work. It is recommended that the authors expand it to include a clearer summary of the main results and their significance.

- **Clarification in Lemma 1 (Page 147, left column, mid-page):**

The transition from the assumption

$$\langle f, \Phi^*(u \pm v) \rangle = 1$$

to the conclusion

$$\langle f, \Phi^*(v) \rangle = 0$$

requires a more detailed justification.

- **Inclusion of Examples:**

Since the paper is relatively short, it would be advisable to suggest that the authors include examples illustrating applications of the main results to the description of isometries in specific classes of Banach spaces. For instance, finite-dimensional Cartan factors – which satisfy the conditions of Theorem 1 – would be appropriate.

The paper presents a rigorously executed study on a relevant and timely topic and may be of interest to specialists in the geometry of Banach spaces, operator algebras, and the mathematical foundations of quantum theory. The content meets the level of a specialized mathematical publication.

It is recommended for publication in the journal *Siberian Electronic Mathematical Reports* after minor revision, addressing the noted technical inaccuracies, improving the abstract, and clarifying certain logical steps in the argumentation.