Report on the Paper

"A SHARP UPPER BOUND FOR THE LENGTH OF INCIDENCE ALGEBRAS"

by

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The paper is devoted to the length computation for incidence algebras. The main result of the paper provides a new upper bound for its length. As a consequence, length is connected to the width and height of the poset.

The results presented are original. The paper meets the standards of the journal. I am pleased to recommend it for publication in the Siberian Electronic Mathematical Reports, although I have some suggestions that the author should consider.

(1) Let $M_n(\mathbb{F})$ be the algebra of matrices over a field $\mathbb{F}$. In 1984, Paz conjectured that $\ell(M_n(\mathbb{F})) = 2n - 2$, for any field $\mathbb{F}$. In the same paper, Paz provided an upper bound for $\ell(M_n(\mathbb{F}))$, which proved his own conjecture for $n \leq 4$. In 2018, Guterman, Laffey, Markova, and Šmigoc showed that $l(\mathcal{S}) = 2n - 2$ whenever a generating set $\mathcal{S}$ of $M_n(F)$ contains a matrix with a minimal polynomial of degree $n - 1$ or $n$. In 2019, Shitov obtained another upper bound for $\ell(M_n(\mathbb{F}))$, and he also proved Paz's conjecture for $n = 5$. In 2025, Khrystik and Maksaev proved Paz's conjecture for $n = 6$ and also showed that $12 \leq \ell(M_n(\mathbb{F})) \leq 13$; and Shitov showed an almost immediate proof of the conjecture with $n = 7$. Thus, some key results on this problem are in these references and the author could mention them all, within [5, 10, 20].

(2) The paper "The length function and matrix algebras" of Markova provides examples of length computation for certain algebras, in particular, for the following classical matrix subalgebras: the algebra of upper triangular matrices, the algebra of diagonal matrices, the Schur algebra, Courter's algebra, and for the classes of local and commutative algebras. I consider it to be an important reference for the

reader interested in the lengths of matrix algebras, so the author could mention after “includes various matrix algebras”.

(3) The author could also mention the most recent papers by Guterman and Kudryavtev on non-associative algebras, after “non-associative algebras”, namely “Algebras of slowly growing length”, “Steady growth of length functions and Malcev algebras” and “The length of Jordan algebras and beyond”.

(4) It is better to write the next equality centered

$$(V^{-1})_{ni} = (-1)^{i+n}.\frac{1}{\det V}.\det\mathbf{V}(\alpha_1, \ldots, \alpha_{i-1}, \alpha_{i+1}, \ldots, \alpha_n) \neq 0,$$

in the proof of Lemma 1.

(5) It might be more interesting for Section 4 to be “Suitable pair of numbers” and Section 5 to be “The case of complete multipartite posets”. If the author agrees with this order, the new Section 4 could have the following structure:

- Definition 9;
- Example(s) that (does not) satisfy Conditions 1 – 6;
- Lemma 8;
- Proposition 4.

(6) In the fifth line of the proof of Lemma 8, check if the correct index is $n$ instead of $n-1$.

(7) The author could update references [7] and [8], as such articles have already been published in Proceedings of the Jangjeon Mathematical Society and Advanced Studies in Contemporary Mathematics, respectively.