Review of the article

by S.N. Antontsev, I.V. Kuznetsov and S.A. Sazhenkov

"Weak solutions of the Navier–Stokes equations
with a short-term intense initial pulse",
submitted for publication in Siberian Electronic Mathematical Reports

In a peer-reviewed article by S.N. Antontsev, I.V. Kuznetsov and S.A. Sazhenkov, a system of Navier-Stokes equations is considered which models the dynamics of a homogeneous viscous incompressible fluid in the presence of a short-term intense pulse that occurs at the initial moment of time. The system contains a positive integer parameter n and it is assumed that the pulse action occurs over a short period of time $(0, 1/n)$. Existence of a weak Leray-Hopf solution to the initial boundary value problems for this system in the case when the pulse duration is fixed, that is, when $n = \text{const}$, is guaranteed by the well-known theory of Navier-Stokes equations. The authors of the article carry out and justify on a strict mathematical level the limit transition in the problem as $n \to +\infty$, that is, when the pulse duration tends to zero. In this case, it is assumed that the total impact of the pulse remains constant. In order to justify the limiting transition, a system of n-uniform estimates is preliminarily constructed for a family of weak Leray-Hopf solutions to the initial problem. As a result of the limit transition, the authors prove that the family of weak Leray-Hopf solutions of the problem under consideration has a subsequence converging to a weak solution of the initial boundary value problem for a system of classical Navier-Stokes equations supplemented by an "adjusted" initial velocity field inheriting complete information on the profile and cumulative effect of an initially non-instantaneous pulse. The corrected initial velocity field is defined as the solution of an additional limiting system of equations for an inviscid fluid obtained on a microscopic time scale corresponding to the pulse duration. At the end of the article, the authors identify two special cases in which the limit system, derived on a microscale of time, can be solved explicitly, which leads to an explicit algebraic expression of the adjusted initial velocity.

The work consists of six sections. The first section provides a statement of the original initial boundary value problem, a review of the literature on the subject of the study, and briefly possible applications of the results obtained in the article are indicated. In the second section, the facts well-known in the theory of Navier-Stokes equations about the solvability of the initial formulation are presented and, in the form of Proposition 2, a statement about uniform (in terms of n) estimates for a family of weak solutions is formulated. Main results of the work related to the limit transition in n are given in the third section. They are formulated in the form of Theorem 1. The fourth and fifth sections are devoted to the substantiation of Proposition 2 and Theorem 1, respectively. Finally, in the sixth section, some remarks are made about increasing the regularity of the solution of the limit model, about the uniqueness of the solution, and, as already mentioned above, two special cases are highlighted in which the expression of the adjusted initial velocity can be set explicitly.

The results obtained by the authors are correct and original. They significantly complement the theory of Navier-Stokes equations in terms of describing the pulsed flow regimes of a viscous incompressible fluid. The work is written in clear scientific language. There are no comments and corrections on the text.

Based on the above, I believe that the article by S.N. Antontsev, I.V. Kuznetsov and S.A. Sazhenkov will be of interest to a wide range of specialists in the field of boundary value problems for viscous fluid equations and I recommend this article for publication in Siberian Electronic Mathematical Reports.

The reviewer