Referee report on the manuscript entitled
"Double total coalition in graphs'
by H. Golmohammadi, S. Alikhani, D. Bakhshesh

The presented article is devoted to a new direction in domination theory in graphs: the study of coalition partitions of graph vertices. Two non-dominating sets form a coalition in a graph if their union is a dominating set. Any set of a coalition partition of graph vertices forms a coalition with some set of the partition. By adding an additional property that a dominating set must satisfy, a wide variety of domination can be defined. The authors study the properties of coalition partitions for double total domination (every vertex of a graph must have at least two neighbors in the dominating set).

The paper answers questions that are already standard in this kind of research: the existence of coalition partitions in graphs, estimating the number of coalitions for a set of partition, finding graphs with extreme values of the coalition number, and calculation the coalition numbers for some graphs.

In my opinion, this article is of interest to specialists in discrete mathematics and can be published after minor corrections (mainly misprints and definitions).

Comments.

1. Page 2. The "k-tuple total domination set"should be defined.

2. Page 3. Definition 2. The last part of the first sentence may be removed (the first part says that all V_j are already not double total dominating sets).

3. Page 4. The proof of Lemma 1 takes one page. Is it possible to shorten the proof?

For example, let $\{V_1, V_2, \ldots\}$ be a double total coalition partition of G and V_1 forms a coalition with every set $V_2, ..., V_k$. Then there is a vertex $v \in V(G)$ such that v has at most one neighbor in V_1 (v may belong to V_1). The vertex v always exists (otherwise, V_1 is a double total dominating set). If v has one neighbor in V_1, then v must have at least one neighbor in every $V_1, ..., V_k$. This implies $k \leq \deg v \leq \Delta(G)$ and, therefore, V_1 can form at most $\Delta(G) - 1$ coalitions. If v has no neighbors in V_1, then v must have at least two neighbors in every set $V_2, ..., V_k$. In this case $2k - 2 \leq \deg v \leq \Delta(G)$ or $k \leq \Delta/2 + 1$.

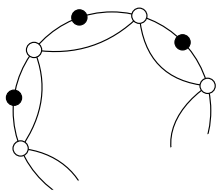
4. Page 6. Line 9 from top: ... each of V_i ... forms ...

5. Page 6. Graph operation "+"should be defined.

6. Page 8. Line 7 from bottom: ... a complete bipartite graph.

7. Page 8. It would be better to insert "then" into the formulation of Theorem 4.

8. Page 8. Theorem 4 [14] is not clear. Consider graph G of (arbitrary large) order n that has $\delta = 2$ and $\Delta = 4$ (see a picture). Since the white vertices form a double total dominating set, the domination number $\gamma_{\times 2,t}(G) \leq n/2$.



9. Page 8. Line 5 in the proof of Theorem 5: ... a double total coalition ...