Referee report on the paper
'On asymptotics of the distribution of the number of upcrossings over
as strip for a random walk with small drift'
by V.I. Lotov

The presented manuscript is a generalisation of the paper [1], where the author has derived a limit theorem for the number of upcrossings assuming that the distribution of increments of the walk is light-tailed. Now he tries to relax his moment assumption in [1] and to prove the same theorem assuming that the moment of order $3 + \delta$ is finite. In my opinion, the main result, Theorem 2, deserves to be published. But I am not really happy with the current version of the proof. In particular, I think that Lemma 1 should be written much more detailed. First, I do not think that one can use ... in mathematical formulae, see (8) and displays following (8). If the author uses Taylor formula then one can use of the classical forms for the remainder. Second, h depends now not only on the drift parameter ε but also on n. Thus, it is not clear how to use arguments from [1]. In the current form of the proof, one can choose any level of truncation and I find this quite strange. Normally, the level of truncation should very appropriately high. This is an additional reason to ask for more details in the proof and for more care when working with many parameters.

I have also a couple of small remarks:

- Is (1) really sufficient for the existence of ρ?

- What does the author mean by the existence of an absolute continuous component **around zero**?

- σ_n^2 appearing in (6) is not defined.

- page 151, line -3: I do not understand how can one take $t = 2\varepsilon k/\sigma_n^2$. We want have fixed t and there is a formula (in the previous line) for fixed k.