

Review of Leila Younsi-Abbaci, Brahmi Boualem, Baya Takhedmit, and Mustapha Moulaï's paper "Optimization of interval fractional functions over efficient set in stochastic integer programming"

In the paper, an algorithm for optimization of an interval fractional function over the efficient set of a stochastic integer problem is described. The authors give a review of papers about optimization over the efficient set. The stated problem is quite interesting because the fractional optimization and stochastic optimization are considered together. However I do not recommend publishing the paper because of the following reasons:

1. This is not clear how the suggested algorithm takes into account the structure of the bound $F_{L_i}(x)$ and $U_{R_i}(x)$ of the objective functions and probabilistic constraints. It seems that the suggested method can be described for any objective function by replacing $G(x)$ in (11). Also, since the chance constraint problem is equivalent to a linear programming problem, the efficient set in (2a) is the efficient set of a linear programming problem with multiple criteria. In fact, a problem of minimization of a nonlinear objective function on the efficient set of a linear programming problem with multiple criteria is solved in the paper. This problem statement is general, but the suggested algorithm is quite obvious because this a relaxation method with successive narrowing of the feasible set. The narrowing of the feasible set is carried out using the known method described in [15].
2. The main problem of the suggested algorithm is solving problems (10) and (11). The objective function is nonlinear but methods for solving these problems are not described. The proposed algorithm could be interesting if methods for solving the auxiliary problems (10) and (11) were proposed.
3. The problem solved is not formulated correctly. The definition of solutions to problems (2a) and (8) is not given. How to understand a solution to an optimization problem with interval coefficients? Moreover, Theorem 1 is cited, but in [22] there is not any theorem. In fact, the problem of minimization of an interval $[F_{L_i}(x), U_{R_i}(x)]$ is replaced by the minimization problem of $F_{L_i}(x) + U_{R_i}(x)$.

4. The statements of problems (1a) and (CCMOSILP) in Section 5 are not correct. The objective functions must be $\mathbb{E} [\sum_{i=1}^n C_i^k x_i]$, where \mathbb{E} is the expectation. This is impossible to define the minimum of a random function, but the expectation of the random function is a deterministic function. Also, the constraints $x_1 + x_2 \leq h_1$ and $x_2 \leq h_2$ must be replaced by $\mathbb{P}(x_1 + x_2 \leq h_1) \geq 1 - \alpha_1$ and $\mathbb{P}(x_2 \leq h_2) \geq 1 - \alpha_2$.
5. In Section 5, the constraints $\mathbb{P}(x_1 + x_2 \leq h_1) \geq 1 - \alpha_1$ and $\mathbb{P}(x_2 \leq h_2) \geq 1 - \alpha_2$ are replaced by $x_1 + x_2 \leq 10.452$, $x_2 \leq 7.3805$. The right sides are not correct. For example, let us check the first one. $\alpha_1 = 0.04$, $\Phi(-1.75) = \alpha_1$. The mean of h_1 is 12, the variance is 3. Thus $12 - \sqrt{3} \cdot 1.75 \approx 8.97 \neq 10.452$
6. Sections 2 and 3 do not contain new results. Deterministic equivalents for stochastic problems and interval arithmetic are well known.
7. In the abstract and in the introduction, it is written that a stochastic programming problems with *joint* chance constraints is solved. The joint chance constraints must be defined as $\mathbb{P}(Tx \leq h) \geq \alpha$. In fact, the problem with constraints (1c) $\mathbb{P}(\sum_{i=1}^n T_{ij}x_i \leq h_j) \geq 1 - \alpha_j$ is solved. These constraints are called *separable* chance constraints. They are easier than the joint ones.
8. In the proof of Theorem 2, Propositions 2.1 and 3.2 are cited, but there are not such propositions in the paper.
9. P. 2. This is now clear why \mathbb{P} is *partially known* probability distribution. According to the paper, the distributions of random variables are known exactly.
10. P. 5. The sense of the notation *Esp* is now clear.
11. P. 8. The first line. The definition of the division is not correct. In the right side of the formula must be the multiplication instead of the division.
12. The sense of brackets in (9) is not clear. They can be omitted.
13. P. 9. Line 4. \hat{x} must be replaced by \hat{x}^1 .

14. P. 10. H_{cs} is introduced as a set, but below H_{cs} is a system of constraints.
15. In the algorithm, G_{inf} and G_{sup} are used but in (11) G_{opt} is used. This is not clear how to find G_{opt} .
16. Algorithm, step 5, problem (R_{cs2}^l) is not described.
17. In (11) $G(x) \leq G_{opt}$ but in the numerical example $G(x) < G_{opt}$ (Step 3, (R_{cs2}^2)).