RESPONSE TO THE REPORT

Dear Sir/Madam,

We would like to express our sincere gratitude to you for carefully reading and thoughtfully reviewing our manuscript. We greatly appreciate the insightful comments and valuable suggestions, which have helped us to improve the clarity and depth of our work. Please find below our detailed responses to the points raised by the esteemed reviewer.

1. Point no. (1) has been incorporated.

2. Point no. (2) has been incorporated.

3. Point no. (3) has been incorporated. Definitions of $\overline{N}(r,f)$, $T(r,f)$ are well known definitions given in [4], [5] [P-2, definition 1.2, 1.3, 1.4, p-23].

4. Regarding point no (4), we have included the description of a_i in (2.1).

5. For point (5), we have used the word 'by no means' and hence the language that was mentioned in the report is correct in the paper.

Regarding suggestion (a), unfortunately, we have not come across any paper or relevant book that provides an example of a function f such that $0 < \rho_2(f) < 1$ in any context. We ourselves are interested in identifying such a function. Any hints, suggestions for constructing such an example, or references indicating the existence of such a function from your end would be greatly appreciated. Further, we see that $\rho_2(e^{e^z}) = 1$ and we think that $0 < \rho_2(e^{e^{\sqrt{z}}}) < 1$, but there is a confusion of having branch point. However, the presence of a branch point in this function introduces additional complexity. Furthermore, any candidate function must satisfy specific sharing conditions, which adds significant difficulty to the construction of such examples and may take considerable time, potentially years to develop. The example given below may be constructed as an illustration of a function of hyper order between 0 and 1. Take an entire function $f(z)$ with the Taylor's series expansion $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and order ρ, then it is known that

$$\lim_{n\to\infty} \frac{\log(\frac{1}{|a_n|})}{n\log(n)} = \frac{1}{\rho}.$$

Using this, we know that, if $f(z) = \sum_{n=0}^{\infty} \frac{z^n}{(n!)^\alpha}$, $\alpha > 0$, then $\rho(f) = \frac{1}{\alpha}$. Now, using Theorem 1.45 in [5], we can say that $\rho(f(z)) = \rho_2(e^{f(z)})$. Combining all the facts we can write that $g(z) = e^{f(z)} = e^{\sum_{n=0}^{\infty} \frac{(e^z)^n}{(n!)^2}}$, has hyper order $\frac{1}{2}$. However, it is not known whether using this function will satisfy the sharing condition stated in Theorem 2.1 of the manuscript. Moreover, we believe that constructing such a function to meet the required conditions is quite tough problem.

We also believe you would agree that, particularly in pure mathematics and especially in complex analysis it is not uncommon for a theory to be developed in a broad and general framework, even when explicit examples satisfying all the conditions are either not yet known or are extremely difficult to construct. Therefore, we humbly and respectfully request that, if no concrete example is currently available, you kindly consider allowing us to retain the existing examples in their present form, and perhaps treat your insightful question as a direction for future research.

Regarding point (b), we observe that the operator $L_j f(z)$ represents a more generalized structure of the function $f(z)$. Indeed, in Examples 2.2 and 2.3, it is evident that the operator $L_j f(z)$ cannot be replaced by an arbitrary operator.
6. According to point (6) we have given the meaning of s in Definition 2.1.
7. Regarding to the point no (7), we have included the meaning of s in Theorem 2.2.
8. Authors are also very interested to find an example for a general case of $0 \leq \rho_2(f) < 1$. If learned reviewer suggest us a way to find such examples, we are ready to accomplish.
9. As suggested in point (9), we have inserted a comma in *Lemma* 3.1, in the second term on the left side of the formula after r.

Further, *Lemma* 3.1 is a very well established lemma, which was used in [2].
10. *Lemma* 3.2 was proved in [3] (See p.109 *Theorem* 2.1 and 2.2).
Lemma 3.3 was proved in [6] (See p.13, lemma 1.3).
Lemma 3.4 was proved in [1] (See p.56, lemma 3.3). We are ready to follow your instructions if you tell us to . But we can not understand how to arrange your comments. Please give us detail.
11. We know that if $T(r, f) \leq T(r, g)$, then $S(r, f)$ can be replaced by $S(r, g)$ As, by the definition of $S(r, f)$ and $S(r, g)$, we can write

$$\frac{S(r,f)}{T(r,f)} \to 0 \text{ and } \frac{S(r,g)}{T(r,g)} \to 0,$$

outside of a set of finite linear measured set. Hence, for $\epsilon > 0$, there exists r_0 such that

$$S(r, f) < \epsilon T(r, f) < \epsilon T(r, g) \text{ for all } r > r_0.$$

Therefore,

$$\frac{S(r,f)}{T(r,g)} \to 0 \implies S(r, f) = o(T(r, g)).$$

In the other way, we have $\frac{T(r,f)}{T(r,g)} = O(1)$. We know that $\frac{S(r,f)}{T(r,f)} \to 0$ as $r \to \infty$ outside a set of finite linear measure. Now

$$\frac{S(r,f)}{T(r,g)} = \frac{S(r,f)}{T(r,f)} \frac{T(r,f)}{T(r,g)} \to 0$$

outside a set of finite linear measure. We have used this argument in *Lemma* 3.5 to replace $S(r, L_j f(z))$ and $S(r, L_j f(z + c))$ by $S(r, f)$.
12. Using the similar argument that was mentioned in **11.** we can say that $S(r, L_j f^{(l)}(z))$ and $S(r, L_j f^{(s)}(z + c))$ also can be replaced by $S(r, f)$.
13. We have changed the '$= S(r, f)$" into '$\leq S(r, f)$', in our manuscript.

Regarding the comment on the English language, we would be grateful if you could kindly specify the exact portions of the manuscript where you found issues. If there are any grammatical errors, typos, or spelling mistakes, we sincerely apologize and would very much appreciate it if you could point them out.

However, we are unsure about the meaning of the remark "we have to proofread the English language." We kindly request that, instead of a general comment, you please identify the specific parts of the manuscript where you believe the language is not appropriate or needs improvement. If possible, we would be thankful for any suggestions on how the concerned passages might be improved. Without such detailed input, it is quite difficult for us to make meaningful changes to the language based solely on a general remark.

The above constitutes our detailed response to your comments, to the best of our ability and as far as practicable. We sincerely hope that our explanations will address your concerns satisfactorily. However, if you have any further suggestions, comments, remarks, or queries, we would be more than willing to respond and provide clarification.

We kindly request that no final decision be made without affording us the opportunity to defend our work. Your understanding and continued engagement are truly appreciated.

With regards,
Authors.

References

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