

Review of the paper of ABHIJIT BANERJEE, JHILIK BANERJEE "Normal and differentiable periodicity of linear shift operators under partial sharing"

In this paper, the periodicity of linear shift operators associated with meromorphic functions under the framework of partial sharing conditions is investigated. The novel concept of differentiable periodicity, which provides a framework for analyzing the behavior of these operators under partial CM or IM sharing is introduced. The findings extend and refine prior results presented in papers of W. J. Chen, Z. G. Huang (2022) and D. C. Pramanik, A. Sarkar (2024). Main results are Theorem 2.1, Theorem 2.2, Theorem 2.3 and Theorem 2.4.

There are many remarks to the work. Let us note just a few.

1) **In Abstract**, remove links "[4] and [12]." Better "in papers of W. J. Chen, Z. G. Huang (2022) and D. C. Pramanik, A. Sarkar (2024)."

2) **Page 1**. In " $\nu_f^a = \dots, \nu_f^\infty = \dots$ " and so on to write " $\nu_f^a(z) = \dots, \nu_f^\infty(z) = \dots$ " and so on.

3) **Page 2**. In Theorem A. "Let f be a finite ordered entire function" replace by "Let f be an entire function of finite order".

Give a definition of $\overline{N}(r, f)$, $S(r, f)$.

4) **Page 3**. In (2.1), clarify what is there a_i ? In the definition L_j it should be noted that it depends on a_j and c .

5) **Page 4**¹. The authors write: "The following example shows that, by no means, the linear shift operator $L_j(z + c)$ in Theorem 2.1, can be replaced by an arbitrary linear shift operator." They give an example (Example 2.2.) for the case $\rho_2(f) = 0$ and a specific operator $\tilde{L}_3 f(z) = 3f(z + 3c) - f(z + 2c) - 2f(z + c) - f(z)$ (although they write: "can be replaced by an arbitrary linear shift operator"). Two issues need to be clarified.

a) Give a general example for $0 \leq \rho_2(f) < 1$ and $\tilde{L}_j f(z)$.

b) Are there linear operators other than the operator $L_j(z + c)$ for which the theorem 2.1 is true?

Similar questions and to Example 2.3.

6) **Page 4**. In the Definition 2.1, specify the meaning of s . Is $s > 1$ a fixed or any integer?

7) **Page 4.** In the Theorem 2.2, specify the meaning of s . Is $s > 1$ a fixed or any integer?

8) **Page 4.** After Note 2.1., the authors write: "The following example shows that, sharing of the non-zero element in Theorem 2.2 can not be dropped." In Example 2.4 $\rho_2(f) = 0$. It is advisable to give an example for a general case $0 \leq \rho_2(f) < 1$.

This is the same remark to the examples 2.5, 2.6, 2.7.

9) **Page 6.** In Lemma 3.1, in the second term on the left side of the formula after r we need "comma".

... = $S(r, f)$ by definition? if not, then what is $S(r, f)$ and specify where in [6] this equality is proven?

10) **Page 6.** Specify where in [5], [15] and [2] the statements of the lemmas 3.2, 3.3 and 3.4 are proved respectively.

11) **Page 6.** The phrase "then $S(r, L_j f(z))$ and $S(r, L_j f(z + c))$ can be replaced by $S(r, f)$ " in the lemma 3.5 needs to be clarified. Where can be replaced?

It is necessary to describe the proof of the lemma 3.5 in more detail.

12) **Page 7.** The lemma 3.6 contains the same remarks as the lemma 3.5.

13) **Page 7.** In the lemma 3.7, it is written ... = $S(r, f)$, but it is proven ... $\leq S(r, f)$

14) **Pages 7, 8.** Verification of the proofs of Lemmas 8, 9 and further review is possible after eliminating the comments stated above.

It is necessary to carefully proofread the text. Especially the English language.

The article requires revision and additional review.

Reviewer