UDK 517.968

UNIQUENESS AND STABILITY OF SOLUTIONS FOR SYSTEMS OF LINEAR INTEGRAL EQUATIONS OF THE FIRST KIND WITH TWO VARIABLES

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Abstract: In this paper is dedicated to the research of the problem of uniqueness and stability of solutions for system of linear integral Equations of the first kind with two variables. Here the operator generated by the kernels is not the compact operator.

Keywords: Linear,system, Inteqral Equations, first kind, solution, uniqueness and stability.

Various issues concerning of integral equations of the first kind were studied in [1-10]. More specifically, fundamental results for Fredholm integral equations of the first kind were obtained in [8], where regularizing operators in the sense of M.M.Lavrent'ev vere constructed for solutions of linear Fredholm integral equations of the first kind. For linear Volterra integral equations of the first and third kinds with smooth kernels, the existence of a multiparameter family of solutions was proved in [9]. The regularization and uniqueness of solutions to systems of nonlinear Volterra integral equations of the first kind were investigated in [5,6].

Inverse and ill-posed problems are currently attracting great interest. Modern methods for solving ill-posed problems are gaining more and more popularity. Results of the inverse problem of microtomography in [11] and numerical methods for solving ill-posed problems in [12] were obtained, inverse problems of biology and medicine [13], of determining the dielectric con stant of a medium, which depends on one spatial variable, based on GPR data in [14].

In the prezent paper, on the basis of the methods of integral transformation, nonnegative quadratic forms and functional analysis the issues of uniqueness and stability and stability of solutions to the system of equations (1).

We consider the system of integral equations

$$Ku = f(t,x), (t,x) \in G = \left\{(t,x) \in R^2 : t_0 \le t \le T, a \le x \le b\right\}, \qquad (1)$$

where

$$Ku \equiv \int\limits_a^b K(t,x,y)u(t,y)dy + \int\limits_{t_0}^T H(t,x,s)u(s,x)ds+ \\ + \int\limits_{t_0}^T \int\limits_a^b C(t,x,s,y)u(s,y)dyds, (t,x) \in G, \tag{2}$$

$$K(t,x,y) = \left\{ \begin{array}{l} A(t,x,y), \ t_0 \le t \le T; \ a \le y \le x \le b; \\ B(t,x,y), \ t_0 \le t \le T, \ a \le x \le y \le b, \end{array} \right\} \tag{3}$$

$$H(t,x,s) = \left\{ \begin{array}{l} M(t,x,s), \ t_0 \le s \le t \le T, \ a \le x \le b; \\ N(t,x,s), \ t_0 \le t \le s \le T, \ a \le x \le b, \end{array} \right\} \tag{4}$$

$A(t,x,y), B(t,x,y), M(t,x,s), N(t,x,s), C(t,x,s,y)$ - known $n \times n-$ dimensional matrix functions defined respectively in the domains

$$G_1 = \{(t,x,y) : t_0 \le t \le T, a \le y \le x \le b\},$$
$$G_2 = \{(t,x,y) : t_0 \le t \le T, a \le x \le y \le b\},$$
$$G_3 = \{(t,x,s) : t_0 \le s \le t \le T, a \le x \le b\},$$
$$G_4 = \{(t,x,y) : t_0 \le t \le s \le T, a \le x \le b\}, G^2 = G \times G,$$

$f(t,x)$ and $u(t,x)-$ respectively known and unknown $n-$ dimensional vektor functions.

We introduce the following notations:
1. We denote by R^n and $M-$ the spaces of all $n-$ dimentional vectors and the spaces of all $n \times n-$ matrix $A = (a_{ij})$ respectively. For vectors

$$u = (u_1, u_2, ..., u_n), v = (v_1, v_2, ..., v_n) \in R^n$$

the scalar product is defined by the formula

$$\langle u, v \rangle = u_1 v_1 + u_2 v_+ ... + u_n v_n.$$

For $u \in R^n$ and $A = (a_{ij}) \in M$ the norms are defined by the formulae

$$||u|| = \sqrt{\langle u, u \rangle}, ||A|| = \left(\sum_{i=1}^n \sum_{j=1}^n (a_{ij})^2 \right)^{\frac{1}{2}};$$

2. let denote by $L_{2,n}(G)-$ the space of all $n-$ dimentional vector functions with elements from $L_2(G)$. In the space $L_{2,n}(G)$ we define the norm as follows

$$||u(t,x)||_{L_2} = \left(\int\limits_{t_0}^T \int\limits_a^b ||u(t,x)||^2 dxdt \right)^{\frac{1}{2}}.$$

3. We assume that $||C(t,x,s,y)|| \in L_2(G^2)$ and $\left[C(t,x,s,y) + C^T(s,y,t,x)\right]$ can be expanded in the series

$$\frac{1}{2}\left[C(t,x,s,y)+C^T(s,y,t,x)\right]$$
$$=\sum_{i=1}^{m}\lambda_i\begin{pmatrix}\varphi_1^{(i)}(t,x)\\ \vdots\\ \varphi_n^{(i)}(t,x)\end{pmatrix}\left(\varphi_1^{(i)}(s,y),...,\varphi_n^{(i)}(s,y)\right),1\le m\le\infty, \tag{5}$$

where $\lambda_1,\lambda_2,...$ are the eigenvalues of the matrix kernel $\frac{1}{2}\left[C(t,x,s,y)+C^T(s,y,t,x)\right]$ numbered in the order of decreasing absolute values $|\lambda_1|\ge|\lambda_2|\ge...$ and $\varphi^1(t,x)=\left(\varphi_\nu^1(t,x)\right),\varphi^2(t,x)=\left(\varphi_\nu^2(t,x)\right)...$ are the corresponding orthonormal sequence of vector eigenfunctions from $L_{2,n}(G)$. We set

$$\left\{\begin{array}{l}P(s,y,z)=A(s,y,z)+B^T(s,z,y),(s,y,z)\in G_1,\\ Q(s,y,\tau)=M(s,y,\tau)+N^T(\tau,y,s),(s,y,\tau)\in G_3.\end{array}\right\} \tag{6}$$

Assume that the following conditions are satisfied:

1) $P^T(s,y,z)=P(s,y,z),(s,y,z)\in G_1,Q^T(s,y,\tau)=Q(s,y,\tau),(s,y,\tau)\in G_3$;
2) $P(s,b,a),Q(T,y,t_0),P_z^{'}(s,b,z),Q_\tau^{'}(T,y,\tau)$ - non-negative matrices respectively for all $s\in[t_0,T],y\in[a,b],(s,z),(\tau,y)\in G$, $||P(s,b,a)||\in C[t_0,T],||Q(T,y,t_0)||\in C[a,b]$, $||P_z^{'}(s,b,z)||\in C(G),||Q_\tau^{'}(T,y,\tau)||\in C(G)$;
3) $P_y^{'}(s,y,a),Q_s^{'}(s,y,t_0),P_{zy}^{''}(s,y,z),Q_{\tau s}^{''}(s,y,\tau)$ - non-positive matrices respectively for all $(s,y)\in G,(s,y,z)\in G_1,(s,y,\tau)\in G_3$, $||P_y^{'}(s,y,a)||\in C(G),||Q_s^{'}(s,y,t_0)||\in C(G),||P_{zy}^{''}(s,y,z)||\in C(G_1)$, $||Q_{\tau s}^{''}(s,y,\tau)||\in C(G_3)$;
4) At least one of the following conditions holds:
   a) the matrix $P_y^{'}(s,y,a)$ - is negative matrix for almost all $(s,y)\in G$;
   b) the matrix $P_z^{'}(s,b,z)$ - is positive matrix for almost all $(s,y)\in G$;
   v) the matrix $Q_s^{'}(s,y,t_0)$ - is negative matrix for almost all $(s,y)\in G$;
   the matrix $Q_\tau^{'}(T,y,\tau)$ - is positive matrix for almost all $(\tau,y)\in G$;
5) The Fredholm operator generated by the matrix kernel $\frac{1}{2}[C(t,x,s,y)+C^T(s,y,t,x)]$ defined by (5) is non negative;
6) The Fredholm operator C generated by the matrix kernel $\frac{1}{2}[C(t,x,s,y)+C^T(s,y,t,x)]$ - defined by (5) is positive.

Theorem 1. Let conditions 1)-5) be satisfied. Then the solution of the system (1) is unique in $L_{2,n}(G)$.

Proof. By virtue (3) and (4), we can write the system (1) in the form

$$\begin{array}{l}\int\limits_a^x A(t,x,y)u(t,y)dy+\int\limits_x^b B(t,x,y)u(t,y)dy+\int\limits_{t_0}^t M(t,x,s)u(s,x)ds+\\ \int\limits_t^T N(t,x,s)u(s,x)ds+\int\limits_{t_0}^t\int\limits_a^b C(t,x,s,y)u(s,y)dyds=f(t,x).\end{array} \tag{7}$$

Multiplying both sides of the system (7) by $u(t,x)$ and integrating over the domain G, we obtain

$$\int_a^b \int_{t_0}^T \left\langle \int_a^y A(s,y,z)u(s,z)dz, u(s,y) \right\rangle dsdy+$$
$$+\int_a^b \int_{t_0}^T \left\langle \int_y^b B(s,y,z)u(s,z)dz, u(s,y) \right\rangle dsdy+$$
$$+\int_a^b \int_{t_0}^T \left\langle \int_{t_0}^s M(s,y,\tau)u(\tau,y)d\tau, u(s,y) \right\rangle dsdy+ \tag{8}$$
$$+\int_a^b \int_{t_0}^T \left\langle \int_s^T N(s,y,\tau)u(\tau,y)d\tau, u(s,y) \right\rangle dsdy+$$
$$+\int_a^b \int_{t_0}^T \left\langle \int_{t_0}^T \int_a^T C(s,y,\tau,z)u(\tau,z)dzd\tau, u(s,y) \right\rangle dsdy =$$
$$= \int_a^b \int_{t_0}^T \langle f(s,y), u(s,y)\rangle dsdy.$$

Hence

$$\int_a^b \int_{t_0}^T \left\langle \int_a^y A(s,y,z)u(s,z)dz, u(s,y) \right\rangle dsdy+$$
$$+\int_a^b \int_{t_0}^T \int_y^b \left\langle u(s,z), B^T s,y,z)u(s,y) \right\rangle dzdsdy+$$
$$+\int_a^b \int_{t_0}^T \left\langle \int_{t_0}^s M(s,y,\tau)u(\tau,y)d\tau, u(s,y) \right\rangle dsdy+$$
$$+\int_a^b \int_{t_0}^T \int_s^T \left\langle u(\tau,y), N^T(s,y,\tau)u(s,y) \right\rangle d\tau dsdy+$$
$$+\int_a^b \int_{t_0}^T \left\langle \int_{t_0}^T \int_a^T C(s,y,\tau,z)u(\tau,z)dzd\tau, u(s,y) \right\rangle dsdy+$$
$$+\int_a^T \int_{t_0}^T \left\langle \int_{t_0}^T \int_y^b C(s,y,\tau,t)u(\tau,z)dzd\tau, u(s,y) \right\rangle dsdy =$$
$$= \int_a^b \int_{t_0}^T \langle f(s,y), u(s,y)\rangle dsdy.$$

The application of Dirichlet's and Fubini's formulas to (8) yields

$$\int_{t_0}^T \int_a^T \left\langle \int_a^y [A(s,y,z) + B^T s,z,y)]u(s,z)dz, u(s,y) \right\rangle dyds+$$

$$+\int_a^y \int_{t_0}^y \left\langle \int_{t_0}^y [M(s,y,\tau)+N^T\tau,y,s)]u(\tau,y)d\tau, u(s,y) \right\rangle dsdy+$$

$$+\int_a^y \int_{t_0}^y \left\langle \int_{t_0}^y \int_a^y \left[C(s,y,\tau,z)+C^T(\tau,z,s,y)\right] u(\tau,z)dzd\tau, u(s,y) \right\rangle dsdy =$$

$$= \int_a^y \int_{t_0}^y \langle f(s,y), u(s,y)\rangle \, dsdy.$$

Here taking into account notation (6), we see that

$$\begin{aligned}
&\int_{t_0}^T \int_a^b \left\langle \int_a^y P(s,y,z)u(s,z)dz, u(s,y) \right\rangle dyds+ \\
&+\int_a^y \int_{t_0}^T \left\langle \int_{t_0}^s Q(s,y,\tau)u(\tau,y)d\tau, u(s,y) \right\rangle dsdy+ \\
&+\int_a^s \int_{t_0}^s \left\langle \int_{t_0}^T \int_a^T \left[C(s,y,\tau,z)+C^T\tau,z,s,y)\right] u(\tau,z)dzd\tau, u(s,y) \right\rangle dsdy = \\
&= \int_a^b \int_{t_0}^b \langle f(s,y), u(s,y)\rangle \, dsdy.
\end{aligned} \tag{9}$$

Let us transform the first two integrals on the left hand side of equation (9).

$$\left\langle Ku, u_y^{'} \right\rangle = \frac{1}{2}(\langle Ku, u\rangle)_y^{'} - \frac{1}{2}\left\langle K_y^{'}u, u \right\rangle; \tag{10}$$

$$\frac{\partial}{\partial \tau} \int_\tau^s u(\xi,y)d\xi = -u(\tau,y). \tag{11}$$

Integrating by parts, and using (10), (11) and the application of Dirichlet's and Fubini's formulas we obtain

$$
\begin{aligned}
&\int\limits_{t_0}^{T}\int\limits_a^b \left\langle \int\limits_a^y P(s,y,z)u(s,z)dz, u(s,y)\right\rangle dyds = \\
&= -\int\limits_{t_0}^{T}\int\limits_a^b \left\langle \int\limits_a^y P(s,y,z)\frac{\partial}{\partial z}\left(\int\limits_z^y u(s,\nu)d\nu\right)dz, u(s,y)\right\rangle dyds = \\
&= -\int\limits_{t_0}^{T}\int\limits_a^b \left\langle P(s,y,z)\left(\int\limits_z^y u(s,\nu)d\nu\right)\Bigg|_a^y - \right. \\
&\left. -\int\limits_a^y P_z^{'}s,y,z)\left(\int\limits_z^y u(s,\nu)d\nu\right)dz, u(s,y)\right\rangle dyds = \\
&= \int\limits_{t_0}^{T}\int\limits_a^b \left\langle \left(P(s,y,a)\int\limits_a^y u(s,\nu)d\nu + \right.\right. \\
&\left.\left.\int\limits_a^y P_z^{'}(s,y,z)\int\limits_z^y u(s,\nu)d\nu\right)dz, u(s,y)\right\rangle dyds = \\
&= \int\limits_{t_0}^{T}\int\limits_a^b \left\langle (P(s,y,a)\left(\int\limits_a^y u(s,\nu)d\nu\right), \left(\int\limits_a^y u(s,\nu)d\nu\right)_y^{'}\right\rangle dyds + \\
&+\int\limits_{t_0}^{T}\int\limits_a^b\int\limits_z^{'} \left\langle P_z^{'}(s,y,z)\left(\int\limits_z^y u(s,\nu)d\nu\right), \left(\int\limits_z^y u(s,\nu)d\nu\right)_y^{'}\right\rangle dydzds = \\
&= \frac{1}{2}\int\limits_{t_0}^{T}\int\limits_a^b \left[\left(\left\langle P(s,y,a)\int\limits_a^y u(s,\nu)d\nu, \int\limits_a^y u(s,\nu)d\nu\right\rangle\right)_y^{'} - \right. \\
&\left. -\frac{1}{2}\left\langle P_y^{'}(s,y,a)\int\limits_a^y u(s,\nu)d\nu, \int\limits_a^y u(s,\nu)d\nu\right\rangle\right] dyds + \\
&+\frac{1}{2}\int\limits_{t_0}^{T}\int\limits_a^b \left\{\int\limits_z^b \left[\left(\left\langle P_z^{'}s,y,z)\int\limits_z^y u(s,\nu)d\nu, \int\limits_z^y u(s,\nu)d\nu\right\rangle\right)_y^{'} - \right.\right. \\
&\left.\left. -\frac{1}{2}\left\langle P_{zy}^{'}(s,y,z)\int\limits_z^y u(s,\nu)d\nu, \int\limits_z^y u(s,\nu)d\nu\right\rangle\right] dy\right\} dzds = \\
&= \frac{1}{2}\int\limits_{t_0}^{T}\left\langle P(s,b,a)\int\limits_a^b u(s,\nu)d\nu, \int\limits_a^b u(s,\nu)d\nu\right\rangle ds - \\
&-\frac{1}{2}\int\limits_{t_0}^{T}\int\limits_a^b \left\langle P_y^{'}(s,y,a)\int\limits_a^y u(s,\nu)d\nu, \int\limits_a^y u(s,\nu)d\nu\right\rangle dyds + \\
&+\frac{1}{2}\int\limits_{t_0}^{T}\int\limits_a^b \left\langle P_z^{'}(s,b,z)\int\limits_z^b u(s,\nu)d\nu, \int\limits_z^b u(s,\nu)d\nu\right\rangle dzds - \\
&-\frac{1}{2}\int\limits_{t_0}^{T}\int\limits_a^b\int\limits_a^y \left\langle P_{zy}^{''}(s,y,z)\int\limits_z^y u(s,\nu)d\nu, \int\limits_z^y u(s,\nu)d\nu\right\rangle dzdyds.
\end{aligned}
\tag{12}
$$

Similarly, for the second integral, we have

$$\int_a^b\int_{t_0}^T\left\langle\int_{t_0}^s Q(s,y,\tau)u(\tau,y)d\tau, u(s,y)\right\rangle dyds =$$
$$= \tfrac{1}{2}\int_a^b\left\langle Q(T,y,t_0)\int_{t_0}^T u(\xi,y)d\xi, \int_{t_0}^T u(\xi,y)d\xi\right\rangle dy-$$
$$-\tfrac{1}{2}\int_a^b\int_{t_0}^T\left\langle Q'_s(s,y,t_0)\int_{t_0}^s u(\xi,y)d\xi, \int_{t_0}^s u(\xi,y)d\xi\right\rangle dsdy+ \tag{13}$$
$$+\tfrac{1}{2}\int_a^b\int_{t_0}^T\left\langle Q'_\tau(T,y,\tau)\int_\tau^T u(\xi,y)d\xi, \int_\tau^T u(\xi,y)d\xi\right\rangle d\tau dy-$$
$$-\tfrac{1}{2}\int_a^b\int_{t_0}^T\int_{t_0}^s\left\langle Q''_{\tau s}(s,y,\tau)\int_\tau^s u(\xi,y)d\xi, \int_\tau^s u(\xi,y)d\xi\right\rangle d\tau dsdy.$$

Substituting (12) and (13) into (9) and taking into account (5), we obtain

$$\tfrac{1}{2}\int_{t_0}^T\left\langle P(s,b,a)\int_a^b u(s,\nu)d\nu, \int_a^b u(s,\nu)d\nu\right\rangle ds-$$
$$-\tfrac{1}{2}\int_{t_0}^T\int_a^b\left\langle P'_y(s,y,a)\int_a^y u(s,\nu)d\nu, \int_a^y u(s,\nu)d\nu\right\rangle dyds+$$
$$+\tfrac{1}{2}\int_{t_0}^T\int_a^b\left\langle P'_z(s,b,z)\int_z^b u(s,\nu)d\nu, \int_z^b u(s,\nu)d\nu\right\rangle dzds-$$
$$-\tfrac{1}{2}\int_{t_0}^T\int_a^b\int_a^y\left\langle P''_{zy}(s,y,z)\int_z^y u(s,\nu)d\nu, \int_z^y u(s,\nu)d\nu\right\rangle dzdyds+$$
$$+\tfrac{1}{2}\int_a^b\left\langle Q(T,y,t_0)\int_{t_0}^T u(\xi,y)d\xi, \int_{t_0}^T u(\xi,y)d\xi\right\rangle dy-$$
$$-\tfrac{1}{2}\int_a^b\int_{t_0}^T\left\langle Q'_s(s,y,t_0)\int_{t_0}^s u(\xi,y)d\xi, \int_{t_0}^s u(\xi,y)d\xi\right\rangle dsdy+ \tag{14}$$
$$+\tfrac{1}{2}\int_a^b\int_{t_0}^T\left\langle Q'_\tau(T,y,\tau)\int_\tau^T u(\xi,y)d\xi, \int_\tau^T u(\xi,y)d\xi\right\rangle d\tau dy-$$
$$-\tfrac{1}{2}\int_a^b\int_{t_0}^T\int_{t_0}^s\left\langle Q''_{\tau s}(s,y,\tau)\int_\tau^s u(\xi,y)d\xi, \int_\tau^s u(\xi,y)d\xi\right\rangle d\tau dsdy+$$
$$+\sum_{i=1}^m\lambda_i\int_a^b\int_{t_0}^T\left|\left\langle\varphi^{(i)}(s,y), u(s,y)\right\rangle\right|^2 dsdy =$$
$$= \int_a^b\int_{t_0}^T\langle f(s,y), u(s,y)\rangle dyds.$$

Suppose that $f(t,x)\equiv 0$ for $(t,x)\in G$. Then, taking into account conditions $1)-5)$, from (14) we see that $u(t,x)=0$ for all $(t,x)\in G$ The theorem 1 is proved.

The family of well-posedness depending on the parameter α, is defined as:

$$M_\alpha = \left\{ u(t,x) \in L_{2,n}(G) : \sum_{\nu=1}^{\infty} \lambda_\nu^{-\alpha} \left| u^{(\nu)} \right|^2 \le c \right\},$$

where $c > 0, 0 < \alpha < \infty$,

$$u^{(\nu)} = \int_{t_0}^{T} \int_{a}^{b} \left\langle u(t,x), \varphi^{(\nu)}(t,x) \right\rangle dx dt, \nu = 1, 2, ..., \infty.$$

It is clear that if $u(t,x) \in M_\alpha$, then $||u(t,x)||_{L_2} \le (c\lambda_1^\alpha)^{\frac{1}{2}}$.

Theorem 2. Let conditions 1), 2), 3) and 6). Be satisfied. Then:
a) the solution of the system (1) is unique in $L_{2,n}(G)$;
b) on the set $K(M_\alpha) \subset L_{2,n}(G)$ the inverse $K^{-1}-$ of operator K, is uniformly continuons with Holder exponent $\frac{\alpha}{2+\alpha}$, i.e.

$$\|u(t,x)\|_{L_2} \le c^{\frac{1}{2+\alpha}} \|f(t,x)\|_{L_2}^{\frac{\alpha}{2+\alpha}}, 0 < \alpha < \infty, \tag{15}$$

where $u(t,x) \in M_\alpha, f(t,x) \in K(M_\alpha), K(M_\alpha)-$ is the image of M_α the action of the operator K defined by formula (2).

Proof. a) In this case, the orthonormal sequence of vector eigenfunctions $\left\{\varphi^{(i)}(t,x)\right\}$ is complete in $L_{2,n}(G)$. Therefore (14) implies the uniqueness of the solution to the system (1) in $L_{2,n}(G)$.

b) Let $f(t,x) \in K(M_\alpha)$. Then the system (1) has a solution
$u(t,x) \in M_\alpha$ and it follows from (14) that

$$\sum_{\nu=1}^{\infty} \lambda_\nu \left| u^{(\nu)} \right|^2 \le ||f(t,x)||_{L_2} ||u(t,x)||_{L_2}.$$

On the other hand,

$$\|u(t,x)\|_{L_2}^2 = \sum_{\nu=1}^{\infty} \left(|u^{(\nu)}|^{\frac{2\alpha}{1+\alpha}} \cdot \lambda_\nu^{\frac{\alpha}{1+\alpha}} \right) \frac{|u^\nu|^{\frac{2}{1+\alpha}}}{\lambda^{\frac{\alpha}{1+\alpha}}} \le$$
$$\left[\sum_{\nu=1}^{\infty} \lambda_\nu |u^\nu|^2 \right]^{\frac{\alpha}{1+\alpha}} \left[\sum_{\nu=1}^{\infty} \lambda_\nu^{-\alpha} |u^{(\nu)}|^2 \right]^{\frac{1}{1+\alpha}}.$$

Combihing the last two inequalities gives estimate (15). The theorem 2 is proved.

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