

REFEREE'S REPORT ON THE ARTICLE
BY S.N. ANTONTSEV, I.V. KUZNETSOV, D.A. PROKUDIN,
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THE IMPULSIVE KELVIN-VOIGT EQUATIONS FOR
TWO-COMPONENT MIXTURES,
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The paper is devoted to the multidimensional initial-boundary value problem for the system of Kelvin–Voigt equations of a two-component mixture of viscoelastic fluids. The state of the medium is characterized by the velocity fields \mathbf{v}_i of the constituents and by the pressures π_i . The mathematical model includes the conservation laws for masses and momenta for the constituents which are a generalization to the two-component case of the well-known Kelvin–Voigt system of equations of dynamics of a one-component viscoelastic fluid. These equations have two features, the presence of which mainly determines the novelty and originality of the research. The first of the features is the presence of the terms $\sum_{j=1}^2 \mu_{ij} \Delta_x \mathbf{v}_j$. Unlike the one-component case, where the viscosity coefficient is a scalar, in the two-component case the viscosity coefficients μ_{ij} form matrix \mathbf{M} whose elements characterize viscous friction. More certainly, the diagonal elements of matrix \mathbf{M} are responsible for the viscous friction within each component, and the non-diagonal elements are responsible for the viscous friction between the two components. The case of a diagonal matrix \mathbf{M} is a subject of the classic theory of the Kelvin–Voigt equations of dynamics of one-component viscoelastic fluids. The article deals with much more complicated case of a non-diagonal and non-triangular matrix \mathbf{M} and do not impose any simplifying assumptions on it except the standard requirement of positive definiteness and symmetry. The second feature is the presence of impulsive terms $\gamma \varphi^n(t) \mathbf{v}_i$, which, as $n \rightarrow \infty$, tend weakly* to the the expressions, the Dirac delta-function, which models impulsive source or damping at the initial moment of time. From the physical viewpoint, the impulsive terms $\gamma \varphi^n(t) \mathbf{v}_i$ are related to dilatant (shear-thickening) and pseudoplastic (shear-thinning) fluids. As a rule, incompressible dilatant and pseudoplastic fluids are fluids in which viscosity and velocity change significantly under impulsive loads. In the paper, the case is considered is which the velocities change drastically, but the viscosity values remain constant. In view of this, absorption (in other terms, the velocity damping) occurs in dilatant liquids, which corresponds to a negative value of the coefficient γ . In turn, a sharp increase in velocity occurs in pseudoplastic liquids, i.e. the coefficient γ is positive. In applications, the equations considered in the paper can be used, for example, for modeling of inhomogeneous loose media, since it is known that under the action of impulsive loading, a loose medium exhibits hydrodynamic properties. Indeed, when exposed to seismic shock waves, liquefaction of soils occurs, which leads to collapse of buildings.

It should be noted that the results obtained by the authors are correct and original. They essentially supplement the theory of of the Kelvin–Voigt equations, being a very non-trivial generalization of the case of a single-component fluid.

The text of the article induces the following remark. During justification of

the estimates in Lemmas 1 and 2, the authors do not mention the dependence of the constants of the parameters of the problem. However, this dependence should be specified explicitly.

This remark does not diminish the overall appreciation of the manuscript. The article is well written, with rather detailed and good quality proof for the results.

I recommend the paper “The impulsive Kelvin-Voigt equations for two-component mixtures” by S.N. Antontsev, I.V. Kuznetsov, D.A. Prokudin, and S.A. Sazhenkov for publication in Siberian Electronic Mathematical Reports.