REGULARIZATION AND STABILITY OF SOLUTIONS OF SYSTEM OF LINEAR INTEGRAL FREDHOLM EQUATIONS OF THE FIRST KIND ON A SEMI-AXIS

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In this article, regularizations and stability of solutions of system linear integral Fredholm equations of the first kind are obtained by the methods of functional analysis.

Key words: linear, inteqral equations, first kind, regularization.

В данной работе, с помощью методам функционального анализа получены регуляризации и устойчивости решений систем линейных интегральных уравнений Фредгольма первого рода.

Ключевые слова: линейные, интегральные уравнения, первого рода, регуляризация.

1. Introduction

Inverse and ill-posed problems are currently attracting great interest. The theory and numerical methods for solving inverse and ill-posed problems were studied in [1-18].The notion of correctness in the works of A.N. Tikhonov [1], M.M. Lavrent'ev [2] and V.K. Ivanov [3], different from the classical one, provided a means for studying ill-posed problems and stimulated interest in integral equations with large applied value.

The fundamental results for Fredholm integral equations of the first kind were obtained by M.M. Lavrentiev in [4], [5], where the regularizing operators in the sense of M.M. Lavrentiev.

In [10]-[12], uniqueness theorems were proved and regularization operators in the sense of Lavrent'ev were constructed for systems of linear Volterra and Fredholm integral equations of the third kind.

In [14]-[16] problems of uniqueness and stability of solutions for linear Fredholm integral equations of the first kind were investigated.

In this work, we apply the method of integral transformation to prove regularizations and stability of solutions of system of linear integral Fredholm equations of the first kind in the semi-axis.

Consider of the system of Fredholm linear integral equations

$$Ku \equiv \int_{-\infty}^{a} K(t,s)u(s)ds = f(t), \quad t \in (-\infty, a] \tag{1}$$

$$Ku = f(t), t \in (-\infty, a]. \tag{2}$$

where

$$K(t,s) = \begin{cases} A(t,s), & -\infty < s \le t \le a, \\ B(t,s), & -\infty < t \le s \le a. \end{cases} \tag{3}$$

$$A(t,s)=(a_{ij}(t,s))=\begin{pmatrix} a_{11}(t,s) & a_{12}(t,s) & \dots & a_{1n}(t,s) \\ a_{21}(t,s) & a_{22}(t,s) & \dots & a_{2n}(t,s) \\ \dots & \dots & \dots & \dots \\ a_{n1}(t,s) & a_{n2}(t,s) & \dots & a_{nn}(t,s) \end{pmatrix},$$

$$B(t,s)=(b_{ij}(t,s))=\begin{pmatrix} b_{11}(t,s) & b_{12}(t,s) & \dots & b_{1n}(t,s) \\ b_{21}(t,s) & b_{22}(t,s) & \dots & b_{2n}(t,s) \\ \dots & \dots & \dots & \dots \\ b_{n1}(t,s) & b_{n2}(t,s) & \dots & b_{nn}(t,s) \end{pmatrix},$$

$f(t)=(f_i(t))=(f_1(t),\dots f_n(t))^T$, $\quad u(t)=(u_i(t))=(u_1(t),\dots u_n(t))^T$.

$B(t,s)$ and $A(t,s)$ are the know matrix functions, $f(t)$ is known vector function, $u(t)$ is unknown vector function.

We introduce possible notation:

1. For vectors $u=(u_1,u_2,\dots,u_n),\ \vartheta=(\vartheta_1,\vartheta_2,\dots,\vartheta_n)\in R^n$

   we define the scalar product by the equality

   $$\langle u,\vartheta\rangle=\sum_{i=1}^{n}u_i\vartheta_i \ .$$

2. For

   $A=(a_{ij})-n\times n-$ square matrix and $u=(u_1,u_2,\dots,u_n)\in R^n$ we define the norm

   $$\|A\|=\left(\sum_{i=1}^{n}\sum_{j=1}^{n}|a_{ij}|^2\right)^{\frac{1}{2}},$$

   $$\|u\|=\left(\sum_{i=1}^{n}|u_i|^2\right)^{\frac{1}{2}}.$$

3. Through $L_{2,n}(-\infty,a]$ denote the space of all $n-$ dimensional vector functions $u(t)=(u_1(t),\dots,u_n(t))$ satisfying the condition

   $u_i(t)\in L_{2,n}(-\infty,a]$ for all $i=1,2,\dots,n.$

   For $u(t)=(u_1(t),\dots,u_n(t))\in L_{2,n}(-\infty,a]$ define the norm

   $$\|u(t)\|_{L_{2,n}}=\left(\int_a^b\|u(t)\|^2\,dt\right)^{\frac{1}{2}}.$$

We will assume that

$$\|K(t,s)\|\in(L_2(-\infty,a]\times(-\infty,a]),\ \ \|f(t)\|\in L_{2,n}(-\infty,a].$$

System of equations (1) by virtue of relation (2) can be expressed as

$$\int_{-\infty}^{t} A(t,s)u(s)ds + \int_{t}^{a} B(t,s)u(s)ds = f(t). \tag{4}$$

Both parts of (4) are scalarly multiplied by the u(t)-vector function and, integrating the results on $-\infty < t \le a,$, we obtain

$$\int_{-\infty}^{a}\int_{-\infty}^{t} \langle A(t,s)u(s),u(t)\rangle dsdt + \int_{-\infty}^{a}\int_{t}^{a} \langle B(t,s)u(s),u(t)\rangle dsdt = \int_{-\infty}^{a} \langle f(t),u(t)\rangle dt,$$

$$\int_{-\infty}^{a}\int_{-\infty}^{t} \langle A(t,s)u(s),u(t)\rangle dsdt + \int_{-\infty}^{a}\int_{s}^{a} \langle B^{*}(s,t)u(s),u(t)\rangle dtds = \int_{-\infty}^{a} \langle f(t),u(t)\rangle dt, \tag{5}$$

where B*(s, t) is the transposed matrix to the matrix B(s, t).

Integrating by parts and using the Dirichlet formula we have

$$\int_{-\infty}^{a}\int_{s}^{a} \langle B(s,t)u(t), u(s)\rangle dtds =$$

$$= \int_{-\infty}^{a}\int_{s}^{a} \langle B^{*}(s,t)u(s),u(t)\rangle dtds = \int_{-\infty}^{a}\int_{-\infty}^{t} \langle B^{*}(s,t)u(s),u(t)\rangle dsdt.$$

Then, from (4) we obtain

$$\int_{-\infty}^{a}\int_{-\infty}^{t} \langle [A(t,s)+B^{*}(s,t)]u(s),u(t)\rangle dsdt = \int_{-\infty}^{a} \langle f(t),u(t)\rangle dt. \tag{6}$$

Denote

$$H(t,s) = \frac{1}{2}\left(A(t,s)+B^{\bullet}(s,t)\right), \qquad (t,s) \in G = \{-\infty < s < t \le a\}$$

Then from (5) we obtain

$$2\int_{-\infty}^{a}\int_{-\infty}^{t} \langle H(t,s)u(s),u(t)\rangle dsdt = \int_{-\infty}^{a} \langle f(t),u(t)\rangle dt. \tag{7}$$

where

$$\int_{-\infty}^{a}\int_{-\infty}^{t} H^{2}(t,s)dsdt < +\infty.$$

We introduce a new matrix function $M(t,s)$ as follows

$$M(t,s) = \begin{cases} H(t,s), & -\infty < s \le t \le a, \\ H^{*}(s,t), & -\infty < t \le s \le a. \end{cases} \tag{8}$$

It's clear that $M(t,s) = M^{*}(t,s)$, $(t,s) \in (-\infty,a] \times (-\infty,a]$.

It is easy to verify the validity of the equality

$$\int_{-\infty}^{a}\int_{-\infty}^{a} \|M(t,s)\|^{2} dsdt < +\infty.$$

Then, it is known that

$$M(t,s) = \sum_{i=1}^{\infty} \frac{\varphi_i(t)\varphi_i(s)}{\lambda_i}, \tag{9}$$

where, λ_i - the characteristic numbers of the matrix kernel $M(t,s)$, which are arranged in ascending order of their modules, $|\lambda_1| \le |\lambda_2| \le \dots$ and $\varphi_1(t),\dots,\varphi_n(t)$ the corresponding orthonormal eigen vector-functions.

It is assumed that $M(t,s)$ is a complete kernel and $0 < \lambda_1 \le \lambda_2 \le \dots$. In this case, the solution of the system (1) will be unique in $L_{2,n}(-\infty,a]$.

In what follows, we will assume that all characteristic numbers of the matrix kernel are positive.

For $u(t) = (u_i(t) \in L_{2,n}(-\infty,a]$ we obtain

$$\|u(t)\|_{2,n} = \left(\sum_{i=1}^{n} \int_{-\infty}^{a} |u_i(t)|^2 dt \right)^{\frac{1}{2}} = \left(\int_{-\infty}^{a} \|u(t)\|^2 dt \right)^{\frac{1}{2}} = \left(\sum_{\nu=1}^{\infty} \left|u^{(\nu)}\right|^2 \right)^{\frac{1}{2}},$$

where

$$u^{(\nu)} = \int_{-\infty}^{a} \left\langle u(t), \varphi^{(\nu)}(t) \right\rangle dt = \int_{-\infty}^{a} \left(\sum_{i=1}^{n} u_i(t)\varphi_i^{(\nu)}(t) \right) dt, \qquad (\nu = 1,2,\dots).$$

We distinguish the family of correctness sets depending on the parameter as follows:

$$N_\alpha = \left\{ u(t) \in L_2(-\infty,a] : \sum_{\nu=1}^{\infty} \lambda_\nu^{-\alpha} \left|u^{(\nu)}\right|^2 \le c \right\}, \text{ where } \quad c > 0, 0 < \alpha < \infty,$$

$$u^{(\nu)} = \int_{-\infty}^{a} \left\langle u(t), \varphi^{(\nu)}(t) \right\rangle dt, \quad (\nu = 1,2,..). \tag{10}$$

Let $u(t) = (u_i(t)) \in N_\alpha$. Then

$$2\int_{-\infty}^{a}\int_{a}^{t}\sum_{\nu=1}^{\infty}\lambda_\nu \left\langle \begin{pmatrix} \varphi_1^{(\nu)}(t) \\ \dots\dots \\ \varphi_1^{(\nu)}(t) \end{pmatrix} (\varphi_1^{(\nu)}(s)\dots\varphi_n^{(\nu)}(s)) \begin{pmatrix} u_1(s) \\ \dots\dots \\ u_n(s) \end{pmatrix}, u(t) \right\rangle dsdt = \int_{-\infty}^{a}\left[\sum_{i=1}^{n} f_i(t)u_i(t) \right] dt$$

$$\|u(t)\|_{L_{2,n}}^2 = \sum_{i=1}^{\infty} \left|u^{(\nu)}\right|^2 = \sum_{\nu=1}^{\infty} \lambda_\nu^{\alpha}\lambda_\nu^{-\alpha} \left|u^{(\nu)}\right|^2 = \lambda_1^{\alpha} \left(\sum_{\nu=1}^{\infty} \lambda_\nu^{-\alpha} \left|u^{(\nu)}\right|^2 \right) \le c\lambda_1^{\alpha},$$

$$\|u(t)\|_{L_2,n}^2 \le c\lambda_1^{\alpha}. \tag{11}$$

We will assume that $f(t) \in K(N_\alpha)$. Then the system (1) has a solution $(u_i(t)) \in N_\alpha$ and by virtue of (7), (8) and (9) we have:

$$2\int_{-\infty}^{a}\int_{a}^{t}\sum_{\nu=1}^{\infty}\lambda_\nu \left\langle \begin{pmatrix} \varphi_1^{(\nu)}(t) \\ \dots\dots \\ \varphi_1^{(\nu)}(t) \end{pmatrix} (\varphi_1^{(\nu)}(s)\dots\varphi_n^{(\nu)}(s)) \begin{pmatrix} u_1(s) \\ \dots\dots \\ u_n(s) \end{pmatrix}, u(t) \right\rangle dsdt = \int_{=\infty}^{a}\left[\sum_{i=1}^{n} f_i(t)u_i(t) \right] dt,$$

$$\sum_{v=}^{\infty} 2\int_{-\infty}^{a}\left[\int_{a}^{t}\left\langle u(s),\varphi^{(v)}(s)\right\rangle ds\right]\left\langle u(t),\varphi^{(v)}(t)\right\rangle dt=\int_{-\infty}^{a}\left\langle f(t),u(t)\right\rangle dt .$$

$$\sum_{v=1}^{\infty}\lambda_v\left|\int_{-\infty}^{a}\left\langle u(t),\varphi^{(v)}(t)\right\rangle dt\right|^2=\int_{-\infty}^{a}\left\langle f(t),u(t)\right\rangle dt,$$

$$\sum_{v=1}^{\infty}\lambda_v\left|u^{(v)}\right|^2=\int_{-\infty}^{a}\left\langle f(t),u(t)\right\rangle dt.$$

Hence, using Hölder's inequalities, we have

$$\sum_{v=1}^{\infty}\lambda_v\left|u^{(v)}\right|\le\left\|f(t)\right\|_{L_{2,n}}\cdot\left\|u(t)\right\|_{L_2,n}. \tag{12}$$

On the other side:

$$\left\|u(t)\right\|_{L_{2,n}}^2=\sum_{v=1}^{\infty}\frac{\left|u^{(v)}\right|^{\frac{2\alpha}{1+\alpha}}}{\lambda_v^{\frac{\alpha}{1+\alpha}}}\lambda_v^{\frac{\alpha}{1+\alpha}}\left|u^{(v)}\right|^{\frac{2}{1+a}}\le\left(\sum_{v=1}^{\infty}\frac{\left|u_v\right|^2}{\lambda_v}\right)^{\frac{\alpha}{1+\alpha}}\left(\sum_{v=1}^{\infty}\lambda_v^{-\alpha}\left|u^{(v)}\right|^2\right)^{\frac{1}{1+\alpha}}.$$

Here we have applied Hölder's inequality for $p=1+a, q=\dfrac{(1+\alpha)}{\alpha}$. Taking into account $u(t)\in N_\alpha$ and (11), from the last inequality we have

$$\left\|u(t)\right\|_{L_{2,n}}^2\le c^{\frac{1}{1+\alpha}}\left(\left\|f(t)\right\|_{L_{2,n}}\left\|u(t)\right\|_{L_{2,n}}\right)^{\frac{\alpha}{1+\alpha}}.$$

Hence we obtain the following stability estimate

$$\left\|u(t)\right\|_{L_{2,n}}\le c^{\frac{1}{2+\alpha}}\left\|f(t)\right\|_{L_{2,n}}^{\frac{\alpha}{2+\alpha}},\ 0<\alpha<\infty . \tag{13}$$

Thus, the following theorem has been proved.

Theorem 1. Let the operator M generated by the matrix kernel $M(t,s)$ be positive, where it is defined $M(t,s)$ by formula (8). Then on the set $K(N_\alpha)$ ($K(N_\alpha)$ image when N_α displayed by the operator K) the operator K^{-1}, the inverse of K, is uniformly continuous with the Hölder exponent $\dfrac{\alpha}{2+\alpha}$, i.e. estimate (13) is valid.

Let us show that the solution of the system of equations

$$\varepsilon u(t,\varepsilon)+\int_{-\infty}^{a}K(t,s)u(s,\varepsilon)ds=f(t),\quad t\in(-\infty,a],\ \varepsilon>0 \tag{14}$$

will be regularizing for equation (1) on the set N_α.

Indeed, by making the following substitution into the in system

equations (14)

$$u(t,\varepsilon)=u(t)+\xi(t,\varepsilon),$$

where $u(t)\in N_\alpha$ - solution of systems of equations (1), we obtain

$$\varepsilon\xi(t,\varepsilon)+\int_{-\infty}^{a}K(t,s)\xi(s,\varepsilon)ds=-\varepsilon u(t).$$

Scalarly multiplying the last system of equations by $\xi(t,\varepsilon)$ and integrating from -∞ to a, taking into account (3) and (9), we have:

$$\varepsilon\left\|\xi(t,\varepsilon)\right\|_{2,n}^{2}+\sum_{\nu=1}^{\infty}\lambda_\nu^{-1}\left|\xi_\nu(\varepsilon)\right|^2\le\varepsilon\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|\left|\xi_\nu(\varepsilon)\right|, \tag{15}$$

where $\xi_i(\varepsilon)$-are the Fourier coefficients for the function $\xi(t,\varepsilon)$, according to the orthonormal system $\varphi^{(\nu)}(t)=\left\{\varphi_i^{(\nu)}(t)\right\}$ that is

$$\xi_\nu(\varepsilon)=\int_{a}^{\infty}\langle\xi(t,\varepsilon),\phi_\nu(t)\rangle dt,(\nu=1,2,\ldots)$$

Applying the Hölder inequality for $p=q=\frac{1}{2}$, from (14) we obtain

$$\left\|\xi(t,\varepsilon)\right\|_{L_{2,n}}\le\left\|u(t)\right\|_{L_{2,n}} \tag{16}$$

$$\sum_{\nu=1}^{\infty}\lambda_\nu\left|\xi_\nu(\varepsilon)\right|^2\le\varepsilon\left\|u(t)\right\|_{L_{2,n}}^{2}\le\varepsilon c\lambda_1^{\alpha},\ \varepsilon>0. \tag{17}$$

On the other side

$$\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|\left|\xi_\nu(\varepsilon)\right|=\sum_{\nu=1}^{\infty}\frac{\left|\xi_\nu(\varepsilon)\right|^{\frac{\alpha}{1+\alpha}}}{\lambda_\nu^{\frac{\alpha}{2(1+\alpha)}}}\cdot\lambda_\nu^{\frac{\alpha}{2(1+\alpha)}}\left|u^{(\nu)}\right|^{\frac{1}{1+\alpha}}\left|\xi_\nu(t,\varepsilon)\right|^{\frac{1}{1+\alpha}}\left|u^{(\nu)}\right|^{\frac{\alpha}{1+\alpha}}.$$

Therefore, after applying the generalized Hölder inequality to the right-hand side for

$p=\frac{2(1+\alpha)}{\alpha}, q=2(1+\alpha), m=2(1+\alpha), n=\frac{2(1+\alpha)}{\alpha},$ we have

$$\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|\left|\xi_\nu(\varepsilon)\right| \le \left(\sum_{\nu=1}^{\infty}\lambda_\nu\left|\xi_\nu(\varepsilon)\right|^2\right)^{\frac{1}{p}}\left(\sum_{\nu=1}^{\infty}\frac{\left|u^{(\nu)}\right|^2}{\lambda_\nu^{\alpha}}\right)^{\frac{1}{q}}\left\|\xi(t,\varepsilon)\right\|^{\frac{2}{q}}\left\|u(t)\right\|^{\frac{2}{p}},$$

$$\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|\left|\xi_\nu(\varepsilon)\right| \le \left(\sum_{\nu=1}^{\infty}\left|\xi_\nu(\varepsilon)\right|^2\lambda_\nu^{-1}\right)^{\frac{a}{(1+\alpha)2}}\left(\sum_{\nu=1}^{\infty}\lambda_\nu^{\alpha}\left|u^{(\nu)}\right|^2\right)^{\frac{1}{(1+\alpha)2}}\left\|\xi(t,\varepsilon)\right\|^{\frac{1}{1+\alpha}}\left\|u(t)\right\|^{\frac{\alpha}{1+\alpha}},$$

$$\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|\left|\xi_\nu(\varepsilon)\right| \le \left(\sum_{\nu=1}^{\infty}\left|\xi_\nu(\varepsilon)\right|^2\lambda_\nu^{-1}\right)^{\frac{1}{p}}\left(\sum_{\nu=1}^{\infty}\lambda_\nu^{\alpha}\left|u^{(\nu)}\right|^2\right)^{\frac{1}{q}}\left\|\xi(t,\varepsilon)\right\|^{\frac{2}{q}}\left\|u(t)\right\|^{\frac{2}{p}}.$$

Further in force

$u(t)\in N_\alpha$, (16) and (17) from the last inequality we have

$$\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|\left|\xi_\nu(\varepsilon)\right| \le \left(\varepsilon c\lambda_1^{\alpha}\right)^{\frac{1}{p}} c^{\frac{1}{q}}\left(c\lambda_1^{\alpha}\right)^{\frac{p+q}{pq}}.$$

Hence, substituting $p=\frac{2(1+\alpha)}{\alpha}$, $q=2(1+\alpha)$, we get

$$\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|\left|\xi_\nu(\varepsilon)\right| \le c^{\frac{1}{2(1+\alpha)}}(c\lambda_1^{\alpha})^{\frac{1}{2}}(\varepsilon c\lambda_1^{\alpha})^{\frac{\alpha}{2(1+\alpha)}}, \tag{18}$$

$$\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|\left|\xi_\nu(\varepsilon)\right| \le c\lambda_1^{\frac{1}{2(1+a)}}c^{\frac{1}{2}}c^{\frac{\alpha}{2(1+\alpha)}}\lambda_1^{\frac{\alpha}{2}}\lambda_1^{\frac{\alpha^2}{2(1+\alpha)}}\varepsilon^{\frac{\alpha}{2(1+\alpha)}}.$$

$$\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|\left|\xi_\nu(\varepsilon)\right| \le c\lambda_1^{\frac{\alpha(2\alpha+1)}{2(1+\alpha)}}\varepsilon^{\frac{\alpha}{2(1+\alpha)}}. \tag{19}$$

Taking into account (18), from (14) we have

$$\left\|u(t,\varepsilon)-u(t)\right\|_{L_{2,n}} \le c^{\frac{1}{2}}\lambda_1^{\frac{a(2a+1)}{4(1+a)}}\varepsilon^{\frac{\alpha}{4(1+\alpha)}}, 0<a<\infty. \tag{20}$$

Thus proven.

Theorem 2. Let the operator M generated by the matrix kernel $M(t,s)$ be positive and $f(t)\in K(N_\alpha)$. Then estimate (20) is valid, where u(t, ε)-is the solution of the system (14), u(t) is the solution of the system (1) $M(t,s)$ and is determined by formula (9).

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