REFEREE'S REPORT

On Finite groups isospectral to groups with abelian Sylow 2-subgroups

by

M. A. Grechkoseeva and A. V. Vasil'ev

In this paper, all groups are finite. The spectrum of a group is the set of orders of its elements. If G is a group, we write $\omega(G)$ for the spectrum of G. We say H is *isospectral* to G if $\omega(G) = \omega(H)$. The group G is said to be *recognizable (by spectrum)* if every group that is isospectral to G is isomorphic to G, it is said to be *almost recognizable* if there are finitely many pairwise nonisomorphic groups that are isospectral to G, and it is said to be *unrecognizable* if there are infinitely such groups.

In this paper, the authors look at the question of recognizability of direct products of copies of a nonabelian simple group whose Sylow 2-subgroup is aabelian. It is known that the nonabelian simple groups with an abelian Sylow 2-subgroup are $L_2(q)$ where q is a prime power that is congruent to 3 or 5 mod 8 or q is a power of 2, ${}^2G_2(q)$ where $q = 3^\alpha$ for $\alpha \geq 3$ odd, or J_1.

It is previously know that $L \times L$ is recognizable when L is a nonabelian simple group with an abelian Sylow 2-subgroup if and only if L is either ${}^2G_2(q)$ or J_1. In particular, if $L \cong L_2(q)$, then it is known that $L \times L$ is unrecongnizable. It is also known that if $k \leq l$ and L^k is unrecognizable, then L^l is unrecongnizable.

In this paper, the authors prove that L^k is unrecognizable when $L \cong {}^2G_2(q)$ and $k = 3$ and when $L \cong J_1$ and $k = 4$. Hence, all cases for L^k are settled when L is nonabelian simple with an abelian Sylow 2-subgroup except for the case when $L = J_1$ and $k = 3$ which the authors leave as an open question.

The work in this paper is new and nontrivial. It is interesting and the the exposition is good. Thus, I recommend publication.