Author's Replies to Referee Report

Dear Editors,

We thank the reviewer for his/her careful reading of our manuscript and the positive assessment of the technical correctness and clarity of our work.

Referee Comments

Comment 1. *The motivation for studying $N_p f = (I^p)^* I^p$ is not clearly explained.*

Response: We had included a brief motivation in the introduction in the original manuscript, where we mentioned that the normal operator N^k_m, being an averaging operator, represents a better measurement model than the momentum ray transform itself. The normal operator for momentum ray transforms is a generalization of the normal operators for the usual ray transforms $I = I^0_m$, see Theorem 2.12.2 of [8], where the notation $\mu^m I$ is used instead of N^0_m.

Comment 2. *It would strengthen the paper if the authors could provide some insight into which components of a tensor f can be recovered from the data $(N_0 f, \dots, N_k f)$ for $k < m$.*

Response: An m tensor field f can be uniquely recovered from the *complete* data $(N^0_m f, \dots, N^m_m f)$ by an explicit inversion formula, it is the main result of our previous work [4]. In the case of the *partial* data $(N^0_m f, \dots, N^r_m f)$ $(r < m)$, no component of f can be recovered. The tensor field $W^r_m f$ is the *full local* information on f which can be recovered from the data $(N^0_m f, \dots, N^r_m f)$ $(r < m)$. This statement is explicitly written in the abstract and at the beginning of page 3. The main result of the current work is the inversion formula expressing $W^r_m f$ through $(N^0_m f, \dots, N^r_m f)$.

Comment 3. *It will be good if the author provides some application of partial momentum ray transform as well.*

Response: We thank the referee for this thoughtful suggestion. The following paragraph is added on page 2 after formula (1):

Momentum ray transforms are used as the main tool in the study of higher order versions of the Calderón inverse problem, see [1,5,2].

This gives the motivation at least for the *complete* data $(I^0_m f, \dots, I^m_m f)$ for an m tensor field f. As far as the *partial* momentum ray transform $(I^0_m f, \dots, I^r_m f)$ $(r < m)$ is concerned, so far we do not know any good application. Nevertheless, the *partial* Saint Venant operator $W^r_m f$ $(r < m)$ has an important application. We added the following sentence on page 2 after formula (3):

Quite similarly, $W^r_m f = 0$ **is the consistency condition for the equation** $d^{r+1}v = f$**, see Theorem 2.17.2 of** [8]**.**

We once again thank the referee for his/her constructive remarks and the recommendation for publication.

Sincerely,
Shubham R. Jathar, Manas Kar, Venkateswaran P. Krishnan, and Vladimir A. Sharafutdinov