Finite Rank Operator in Non-Archimedean Sequence Spaces

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Abstract. In this paper, we give a characteristic properties for operators of finite rank in the non-Archimedean sequence spaces. Using a technical lemma, we studied the spectral theory for this class of linear operators.

Key words: Non-Archimedean sequence spaces, Finite rank operators, Spectral theory.

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1 Introduction

The theory of linear operators in non-Archimedean $(n.a.)$ Banach space and locally convex space over $n.a.$ field $\mathbb{K}$ has been treated by several authors.
In [7], Monna succeeded in proving that the Banach-Steinhaus theorem for locally convex spaces remains true for non-Archimedean case.
Serre [9] studied completely continuous linear operator. He demonstrates that we can apply Fredholm's theory to the completely continuous linear operator in free $n.a.$ Banach spaces

In [4], El amrani, Blali and Taybi introduce a spectral analysis for finite rank perturbation of diagonal operator in non-Archimedean Banach space of countable type and in [5], they introduce a spectral analysis for compact and self-adjoint perturbation of diagonal operator in non-Archimedean Banach space of countable type.

In this work, we introduce and study properties of the so-called finite rank operators

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in the non-Archimedean sequence space $E(X)$, where X is a non-Archimedean locally $\mathbb{K}-$convex space. Namely, we show that all finite rank operator F can be written in the following form $F = \sum_{k=1}^{m} x_k \otimes y_k$ where $x_k \in E(X)^\beta$ and $y_k \in E(X)$ for $k = 1, ..., m$.

And under some appropiate assumption, we will show that the spectrum $\sigma(F)$ of the finite rank operator is given by

$$\sigma(F) = \{\lambda \in \mathbb{K} \setminus \{0\} \ : \ \det P(\lambda) = 0\} \cup \{0\},$$

where $P(\lambda)$ is the $m \times m$ square matrix given by $P(\lambda) = (a_{ij}(\lambda))_{i,j=1...m}$ with $a_{ij}(\lambda) = \lambda\delta_{ij} - \sum_{n=1}^{\infty} \langle x_n^i, y_n^j \rangle$, with $x_k = (x_n^k)_n \in E(X)^\beta$, $y_k = (y_n^k)_n \in E(X)$ for $k = 1...m$.

2 Preliminaries

Throughout this paper $\mathbb{K}$ denotes a non trivial field which is complete with respect to a non-Archimedean valuation denoted $|.|$, X and Y are two locally $\mathbb{K}-$convex spaces that are in separated duality $\langle X, Y \rangle$. The duality theory for locally $\mathbb{K}$-convex spaces can be found more extensively in [1] and [11].

A nonempty subset A of a $\mathbb{K}-$vector space X is called $\mathbb{K}-$convex, if $\lambda x + \mu y + \gamma z \in A$, whenever $x, y, z \in A$, $\lambda, \mu, \gamma \in \mathbb{K}$, $|\lambda| \leq 1$, $|\mu| \leq 1$, $|\gamma| \leq 1$, and $\lambda + \mu + \gamma = 1$. A is said to be absolutely $\mathbb{K}-$convex, if $\lambda x + \mu y \in A$, whenever $x, y \in A$, $\lambda, \mu \in \mathbb{K}$, $|\lambda| \leq 1$, $|\mu| \leq 1$, and $\lambda + \mu = 1$. For a nonempty set $A \subset X$, its absolutely $\mathbb{K}-$convex hull $c_0(A)$ is the smallest absolutely $\mathbb{K}-$convex set that contains A. If A is a finite set $\{x_1, \dots, x_n\}$, we sometimes write $c_0(x_1, \dots, x_n)$ instead of $c_0(A)$.

A sequence $(e_i)_i$ is a Schauder basis for X, if every $x \in X$ can be written uniquely as $x = \sum_{i=1}^{\infty} \lambda_i x_i$, and the coefficient functionals $f_j : x \mapsto \lambda_j$ are continuous.

A base $(e_i)_i$ of X is called an orthogonal base of X if the topology of X can be defined by a family $(\mathcal{P})$ of semi-norms $n.a.$ satisfying the following condition:

$$x = \sum_i \lambda_i e_i \in X \Rightarrow p(x) = \sup_i (\lambda_i e_i) \ \ for \ all \ p \in (\mathcal{P}).$$

• $(\omega(X), \tau_\omega(X))$ is the linear space of all sequences in X endowed with the product topology $\tau_\omega(X)$, which is generated by the family of $n.a.$ seminorms $(p_n)_{n\in\mathbb{N}, p\in(\mathcal{P})}$, where, for all $n \in \mathbb{N}$, all $\overline{x} = (x_n)_n \in \omega(X)$ and all $p \in (\mathcal{P})$, $p_n(\overline{x}) = p(x_n)$, where $(\mathcal{P})$ is a family of $n.a$ seminorms which define the topology of locally $\mathbb{K}-$convex space X; this space is noted $\omega(\mathbb{K})$ (or ω, for short) in case when $X = \mathbb{K}$.

- A sequence space over X is a subspace of $\omega(X)$.
- We define the sequence space $\varphi(X)$ over X by:

$\varphi(X) = \{(x_k)_k \in \omega(X) : there\ exists\ k_0 \in \mathbb{N} :\ x_k = 0\ for\ all\ k \geq k_0\}$.

- If $A \subset \omega(X)$, the β-dual of A is the subspace of $\omega(Y)$, which is defined by $A^\beta = \{(y_n)_n \in \omega(Y) : \lim_n \langle x_n, y_n\rangle = 0\ for\ all\ (x_n)_n \in A\}$; we define B^β, if $B \subset \omega(Y)$, in the same way. A is called perfect, if $A^{\beta\beta} = A$. For all $A \subset \omega(X),\ A^\beta$ is perfect.

if A is perfect, then $\varphi(X) \subset A$.

- Let $E(X)$ and $E(Y)$ be two sequence spaces on X and Y, respectively, such that $E(Y) \subset E(X)^\beta$, we define on the pair $(E(X), E(Y))$, the following duality $\langle (x_n)_n, (y_n)_n\rangle = \sum_{n=1}^{\infty} \langle x_n, y_n\rangle$ for all $(x_n)_n \in E(X)$ and all $(y_n)_n \in E(Y)$. We consider this duality such that $\varphi(X) \subset E(X)$ and $\varphi(Y) \subset E(Y)$, this duality is separated. For all $j \in \mathbb{N}$, we define the linear map $\delta_j^X : X \to E(X),\ x \mapsto \delta_j(x)$, where $\delta_j(x)$ is the sequence with x in the j-th place and 0 elsewhere.

For every $y = (y_n)_n \in E(X)^\beta$, we consider $w_y(x) = \sup_n |\langle x_n, y_n\rangle|$ for every $x = (x_n)_n \in E(X)$, then w_y is $n.a$ seminorm over $E(X)$. Let Na the topology over $E(X)$ generated by the family of $n.a$ seminorms $(w_y)_{y \in E(X)^\beta}$. Na is the natural topology studied by De Grande De-Kimpe in the particular case of $n.a$ scalar sequences [2] and by El amrani, Ameziane Hassani and Babahmed in [3] in the general case. The topology Na on $E(X)$ is a polar and solid topology. It is also the coarest solid and polar topology on $E(X)$. If $E(X)$ is perfect, then the topology Na on $E(X)$ is compatible with the duality $\langle E(X), E(X)^\beta\rangle$. In [3] El Amrani, Ameziane Hassani and Babahmed pove that if $E(X)$ is perfect and $(e_n)_n$ is a Schauder basis of $X_\sigma = (X, \sigma(X, Y))$, then for every polar, solid and compatible topololy τ with the duality $\langle E(X), E(Y)\rangle$, $(\delta_j(e_n))_{n,j}$ is a Schauder basis of $(E(X), \tau)$.

- The set of all continuous linear operators of X in itself is denoted by $L(X)$.
- Let $T \in L(X)$, then:

$*$ T is called an operator of finite rank if $R(T)$, the range of T, is a finite dimensional subspace of X. The set of all finite rank operators of X is denoted by $\mathcal{F}(X)$.

$*$ A subset A of a locally $\mathbb{K}-$convex space X is compactoid, if for each neighbourhoud U of zero, there exist $x_1, ..., x_n \in X$ such that $A \subset U + c_0(x_1, ..., x_n)$.

$*$ T is called an operator compact if there exists a neighbourhood U of 0 in X such that $T(U)$ is compactoid. The set of all compact operators of X is denoted by $C(X)$.

3 Main Results

In this section we present a study for the theory of finite rank operators in the non-Archimedean sequence spaces.

Theorem 3.1. *Let X and Y be locally convex spaces over $\mathbb{K}$. Suppose that the topologies of X and Y are determined respectively by the families of continuous $n.a$ semi-norms Γ and Γ'. Let T be a linear map from X to Y. Then, T is continuous if and only if for all $q \in \Gamma'$ there exists $p \in \Gamma$ and $M > 0$ such that $q(T(x)) \leqslant Mp(x)$ for all $x \in X$.*

Proof. see [6] or [11]. □

For all $x \in E(X)^\beta$ and all $y \in E(X)$, we define an application on $E(X)$ in the following way:

$$\begin{aligned} x \otimes y : E(X) &\to E(X) \\ z &\mapsto (x \otimes y)(z) = \langle x, z \rangle . y \end{aligned}$$

Theorem 3.2. *If $E(X)$ is perfect and $(e_n)_n$ is a Schauder basis of $X_\sigma = (X, \sigma(X, Y))$ then, any linear operator $T : E(X) \to E(X)$ can be written in the form*

$$T = \sum_j \sum_n \sum_i a^j_{in} \delta^*_j(e_n) \otimes \delta_j(e_i).$$

Proof. According to theorem 1 of [3], $(\delta_j(e_n))_{n,j}$ is a Schauder basis of $(E(X), Na)$ and $(\delta^*_j(e_n))_{n,j}$ is the dual basis, associated with $(\delta_j(e_n))_{n,j}$, then for all $n \in \mathbb{N}$, we have $T(\delta_j(e_n)) = \sum\limits_j \sum\limits_i a^j_{in} \delta_j(e_i)$ with $a^j_{in} \in \mathbb{K}$. Then for all $x = \sum\limits_j \sum\limits_n x^j_n \delta_j(e_n) \in E(X)$ see [3], and

$$T(x) = \sum_j \sum_n x^j_n T(\delta_j(e_n)),$$

then,

$$\begin{aligned} T(x) &= \sum_j \sum_n \sum_i x^j_n a^j_{in} \delta_j(e_i) \\ &= \sum_j \sum_n \sum_i a^j_{in} \langle \delta^*_j(e_n), x \rangle \delta_j(e_i) \\ &= \sum_j \sum_n \sum_i a^j_{in} (\delta^*_j(e_n) \otimes \delta_j(e_i))(x) \end{aligned}$$

Consequently, $T = \sum\limits_j \sum\limits_n \sum\limits_i a^j_{in} \delta^*_j(e_n) \otimes \delta_j(e_i)$. □

Let's start with the following example of sequence spaces which admits an othogonal base.

Example 3.1. *Let Λ a $n.a$ scalar sequence space (a subspace of the vector space ω), endowed with Na the natural (or normal) topology generated by the family of $n.a.$ semi-norms $(w_y)_{y\in\Lambda^\beta}$ (Λ^β is also called the Köthe dual of Λ). Perz-Garcia and Schikhof prove that, in [8] Theorem 9.4.3, the sequence $e_1, e_2, \ldots,$ of the unit vectors of ω, is an orthogonal base of (Λ, Na).*

Remarks 3.1.

1. *The previous theorem is also true if (X, τ) has a Schauder basis, for all topology τ on X.*
2. *The previous theorem is true in $(E(X), \tau)$, for all topology τ that is polar, solid and compatible with the duality $< E(X), E(Y) >$ where X and Y are two $n.a$ topological vector spaces over $\mathbb{K}$ that are in separated duality $< X, Y >$ such that $X_\sigma = (X, \sigma(X, Y))$ has a Schauder basis and $E(Y) \subset E(X)^\beta$.*

Throughout the rest of this paper, unless explicitly stated otherwise, the space $(E(X), Na)$ is assumed to have an orthogonal basis.

Theorem 3.3. *Let $(E(X), Na)$ be a sequence space over X, then $T : E(X) \to E(X)$ is a finite rank operator if and only if there exist two sequences $x_1, \ldots, x_m \in E(X)^\beta$ and $y_1, \ldots, y_m \in E(X)$ such that $T = \sum_{k=1}^{m} x_k \otimes y_k$.*

Proof. Suppose that $T = \sum_{k=1}^{m} x_k \otimes y_k$ and show that $T \in \mathcal{F}(E(X))$.
It is clear that T is a linear operator with rank $R(T) \subset E(X)$ generated by $\{y_1, \ldots, y_m\}$, so $R(T)$ has dimension at most m. For all $z \in E(X)$ we have

$$Tz = \sum_{k=1}^{m} (x_k \otimes y_k)(z).$$

$$= \sum_{k=1}^{m} \langle x_k, z\rangle y_k$$

Let $\mu = (\mu_k)_k \in E(X)^\beta$ then,

$$w_\mu(Tz) = \sup_k |\langle Tz, \mu_k\rangle|$$

$$
\begin{aligned}
&= \sup_k |\langle \sum_{k=1}^{m} \langle x_k, z\rangle y_k, \mu_k\rangle| \\
&\le \sup_k |\langle x_k, z\rangle \langle y_k, \mu_k\rangle| \\
&\le \sup_k |\langle x_k, z\rangle| \sup_k |\langle y_k, \mu_k\rangle| \\
&\le w_\mu(y).w_x(z).
\end{aligned}
$$

Hence $T \in \mathcal{F}(E(X))$.

Conversely, suppose that $T \in \mathcal{F}(E(X))$ and show that $T = \sum_{k=1}^{m} x_k \otimes y_k$.

Let $\{y_1, \ldots, y_m\} \subset E(X)$ generates $R(T)$. Then, for all $z \in E(X)$,

$$Tz = \sum_{k=1}^{m} \lambda_k(z) y_k.$$

Now for each $k = 1, \ldots, m$, $z \longmapsto \lambda_k(z) = \langle Tz, y_k\rangle$ is a continuous linear funtional on $E(X)$. Therefore, for each $k = 1, \ldots, m$ there exist $x_k \in E(X)^\beta$ such that

$$\lambda_k(z) = \langle Tz, y_k\rangle = \langle x_k, z\rangle \;\; 1 \le k \le m,$$

then we get

$$\sum_{k=1}^{m} \langle x_k, z\rangle y_k = \sum_{k=1}^{m} (x_k \otimes y_k)(z) = Tz.$$

□

Proposition 3.1. *If $A \in L(E(X))$ and $B \in \mathcal{F}(E(X))$, then AB and BA belong to $\mathcal{F}(E(X))$.*

Proof. We write $B = \sum_{k=1}^{m} x_k \otimes y_k$ where $x_1, \ldots, x_m \in E(X)^\beta$ and $y_1, \ldots, y_m \in E(X)$. Now

$$AB = A\left(\sum_{k=1}^{m} x_k \otimes y_k\right) = \sum_{k=1}^{m} A(x_k \otimes y_k) = \sum_{k=1}^{m} (x_k \otimes Ay_k) \in \mathcal{F}(E(X)).$$

Similarly, letting $B = \sum_{k=1}^{m} x_k \otimes y_k$ where $x_1, \ldots, x_m \in E(X)^\beta$ and $y_1, \ldots, y_m \in E(X)$. It follows that,

$$BA = \sum_{k=1}^{m} (x_k \otimes y_k)A = \sum_{k=1}^{m} (x_k A) \otimes y_k \in \mathcal{F}(E(X)).$$

□

The resolvent of a continuous linear operator $T : E(X) \to E(X)$ is defined by

$$\varrho(T) = \{\lambda \in \mathbb{K} : \lambda I - T \; is \; a \; bijection \; and \; (\lambda I - T)^{-1} \in L(E(X))\},$$

where I is identity of $E(X)$. The spectrum $\sigma(T)$ of T is then defined by

$$\sigma(T) = \mathbb{K} \backslash \varrho(T).$$

A scalar $\lambda \in \mathbb{K}$ is called an eigenvalue of T, whenever there exist a nonzero $u \in E(X)$ (called eigenvector associated with λ) such that $Tu = \lambda u$. The collection of all eigenvalues of T is denoted by $\sigma_p(T)$ (called punctual Spectrum) and is defined by

$$\sigma_p(T) = \{\lambda \in \sigma(T) : N(\lambda I - T) \neq \{0\}\}.$$

The index of T is defined by $\chi(T) = \eta(T) - \delta(T)$, where $\eta(T) = \dim N(T)$ and $\delta(A) = \dim(E\,(X)\,/R(T))$.

Lemma 3.1. *If $E(X)$ is perfect, then it is $Na-$sequentially complete.*

Proof. See [3]. □

Lemma 3.2. *Let X be a sequentially complete locally convex spaces. Then for any $T \in C(X)$ we have $I_X + T \in \Phi(X)$ and $\chi(I_X + T) = 0$; in particular, the operator $I_X + T$ is injective if and only if it is surjective.*

Proof. See [10]. □

Lemma 3.3. *Let $E(X)$ be a perfect sequence space over X. Consider in $(E(X), Na)$ the finite rank operator*

$$F = \sum_{k=1}^{m} x_k \otimes y_k,$$

where $x_k \in E(X)^{\beta}$ and $y_k \in E(X)$ for $k = 1, ..., m$, then the operator $I - F$ is invertible if and only if $\det P \neq 0$, where P is the $m \times m$ square matrix given by $P = (a_{ij})_{i,j=1,...,m}$, $a_{ij} = \delta_{ij} - \sum_{n=1}^{\infty} \langle x_n^i, y_n^j \rangle$ for $i, j = 1, ..., m$.

Proof. Using the precedent lemma it follows that the operator $I - F$ is invertible if and only if it is surjective if and only if it is injective ($N(I - F) = \{0\}$). To prove that it

suficient to show that $N(I-F)=\{0\}$ if and only if $\det P \neq 0$.
For that, let $w \in E(X)$ such that $(I-F)w=0$. Equivalently

$$w - \sum_{k=1}^{m} \langle x_k, w\rangle y_k = 0 \qquad (1.1).$$

Now apply $\langle ., .\rangle$ to the equation (1.1) with respectively $x_1, \ldots, x_m \in E(X)^\beta$ for all $k = 1, \ldots, m$; we obtain the following system of equation

$$P \begin{pmatrix} \langle x_1, w\rangle \\ \vdots \\ \langle x_m, w\rangle \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \qquad (1.2).$$

If we suppose that $N(I-F) \neq \{0\}$, then

$$w = \sum_{k=1}^{m} \langle x_k, w\rangle y_k$$

and hence at lent one of the following scalars $\langle x_1, w\rangle, ..., \langle x_m, w\rangle$ is non zero. Consequently the equation (1.1) had at least one non trivial solution which yields $\det P = 0$.
Conversely, if $\det P = 0$, there exist some scalars $\xi_1, ..., \xi_m$ not all zeros, such that with $\xi = (\xi_1, ..., \xi_m)^t$ we have

$$P \begin{pmatrix} \xi_1 \\ \vdots \\ \xi_m \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \qquad (1.2)$$

We take $w = \sum\limits_{k=1}^{m} \xi_k y_k$ and we obtain $(I-F)w = 0$.
Now $w \neq 0$, if not $\langle x_k, w\rangle = \delta_{kj}\xi_j = 0$ for $j = 1, ..., m$ which yields $\xi_j = 0$ for $j = 1, ..., m$ and that contradict the fact that some ξ_j are nonzero unique then $N(I-F) \neq 0$. □

Theorem 3.4. *Let $E(X)$ be a perfect sequence space over X. Consider in $(E(X), Na)$ the finite rank operator*

$$F = \sum_{k=1}^{m} x_k \otimes y_k$$

where $x_k \in E(X)^\beta$ and $y_k \in E(X)$ for each $k = 1, 2, ..., m$. Then the spectrum of F is given by:

$$\sigma(F) = \{\lambda \in \mathbb{K} \setminus \{0\} \ : \ \det P(\lambda) = 0\} \cup \{0\},$$

where $P(\lambda)$ is the $m \times m$ square matrix given by, $P(\lambda) = (a_{ij}(\lambda))_{i,j=1,\dots,m}$ with $a_{ij}(\lambda) = \lambda\delta_{ij} - \sum_{n=1}^{\infty}\langle x_n^i, y_n^j\rangle$ for $i, j = 1, \dots, m$.

Proof. Consider the operator $\lambda I - F$, clearly $\lambda = 0$ is necessairly in the spectrum F is not invertible.
Now suppose $\lambda \neq 0$, then $\lambda I - F = \lambda(I - F_\lambda)$ where $F_\lambda = \lambda^{-1}F$ is a finite rank operator it is clear that $\lambda I - F$ is invertible if and only if $I - F_\lambda$ is invertible, and $(\lambda I - F)^{-1} = \lambda^{-1}(I - F_\lambda)^{-1}$.
Now using the Lemma 3.2 it follows that $I - F_\lambda$ is invertible if and only if $N(I - F_\lambda) = \{0\}$. By using the same idea of the precedent Lemma it follows that $N(I - F_\lambda) = \{0\}$ if and only if $\det P(\lambda) \neq 0$. □

Application: Let $m(\mathbb{K})$ be the set of all bounded sequences in $\mathbb{K}$ and $c_0(\mathbb{K})$ the set of all zero sequences in $\mathbb{K}$ equipped with the $n.a.$ norm $\|(x_i)\|_\infty = \sup_i |x_i|$. Consider the separating duality $\langle c_0(\mathbb{K}), m(\mathbb{K})\rangle$. We have $(c_0(\mathbb{K}), \|.\|_\infty)' = m(\mathbb{K})$, then the topology defined on $c_0(\mathbb{K})$ by the $n.a.$ norm $\|.\|_\infty$ is compatible with the duality $\langle c_0(\mathbb{K}), m(\mathbb{K})\rangle$.
Let $(e_i)_{i\in\mathbb{N}}$ be the canonical basis of $c_0(\mathbb{K})$ and $(e'_i)_{i\in\mathbb{N}}$ is the dual basis, associated with $(e_i)_{i\in\mathbb{N}}$, of dual space $(c_0(\mathbb{K}), \|.\|_\infty)'$. Then,

- Every continuous operator $T : c_0(\mathbb{K}) \to c_0(\mathbb{K})$ can be written in the following form $T = \sum_{ij} \alpha_{ji} e'_i \otimes e_j$ where $\alpha_{ji} \in \mathbb{K}$ for all $i, j \in \mathbb{N}$.
- $T \in \mathcal{F}(c_0(\mathbb{K}))$ if and only if there exist two sequences $x_1, \dots, x_n \in m(\mathbb{K})$ and $y_1, \dots, y_n \in c_0(\mathbb{K})$ such that $T = \sum_{k=1}^{n} x_k \otimes y_k$.
- Consider the finite rank operator $F = \sum_{k=1}^{n} x_k \otimes y_k$ where $x_k \in m(\mathbb{K}),\ y_k \in c_0(\mathbb{K})$ for each $k = 1, 2, ..., n$. Then the spectrum of F is given by:

  $$\sigma(F) = \{\lambda \in \mathbb{K} \setminus \{0\} \ : \ \det P(\lambda) = 0\} \cup \{0\},$$

  where $P(\lambda)$ is the $n \times n$ square matrix given by, $P(\lambda) = (a_{ij}(\lambda))_{i,j}$ with $a_{ij}(\lambda)) = \lambda\delta_{ij} - \langle x_i, y_j\rangle$ for each $i, j = 1, \dots, n$.

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