

Referee Report on “Weakly periodic matrices over algebraically closed fields”

by P. Danchev and A. Pojar

In this article, the authors study a question about certain additive decompositions of elements in matrix rings. Specifically, they are interested in when a matrix over an algebraically closed field can be expressed as the sum of a potent matrix (one that satisfies a polynomial equation of the form $X^n = X$ for some n) and a square nilpotent. This question is connected to ideas that have been considered by classical ring theorists for decades, and the topic is interesting for this reason.

The article under review contains a number of inaccuracies that make it extremely difficult to assess. Some are language, and some are mathematical. I do not intend to provide a complete list, but will list some representative items at the bottom. In sum, however, these sorts of errors lead me to not be able to accurately assess the correctness or impact of the work. The authors will need to make some changes and improve the exposition before this article can be properly refereed.

1. Page 2, Lemma 2.1: It is quite hard to understand what is being said here. I presume that $\alpha_1, \dots, \alpha_k$ are integers; are you saying that their sum is no more than n ? Further, are you saying the roots of unity λ_i are distinct? Because, if not, I am not sure I understand why it must be the case that $k \leq m$.
2. Page 3, Example 2.3: The order in which the variables is quantified here is extremely confusing. You first let l, n, m be integers, but then you later insist that $p^l \leq n - 1$. Also, what do you mean by “the equation $x^m - 1 = 0$ holds over \mathbb{F} ”?
3. Page 4, Example 2.4: If you are working in a field of characteristic 3, then the polynomial $x^{12} - 1$ is equal to $(x^4 - 1)^3$, and so all of your 12-th roots of unity are just 4-th roots of unity, aren't they? This is confusing.
4. Page 5, Lemma 3.1: It seems you could use Jordan canonical form here to simplify matters?