

Dear editors,

enclosed, please, find the revised version of the manuscript "On Runge type theorems for solutions to strongly uniformly parabolic operators" and the point-by-point answers to the questions and the comments by the reviewer.

Sincerely, Alexander Shlapunov, Pavel Vilkov

To Reviewer.

Dear reviewer, we appreciate your efforts to make our paper better.

We corrected all the typos you indicated on pages 3-4 of the report; we also found and corrected some more misprints. The only point is the assumption (α_4) where seemingly a typo from S.D. Eidel'man's paper ([6] in the old version) resurfaced; of course, you are right, the assumption should provide the regularity of the coefficients of the operator \mathcal{L}^* and we corrected the text.

Thank you also very much for the suggested interesting papers related to the field. All of them, that are published, we added to the list of references. Actually, the paper [Ma55] by B. Malgrange was already in the list with some typos in the data ([17] in the old version and [24] in the corrected one). Of course, we knew on the contribution by P.D. Lax and F.E. Browder to the topic but referring to the books by N. Tarkhanov where approximation theorems for elliptic operators were discussed, we tacitly meant also their bibliography. However we agree indeed that this was not appropriate in relation to their outstanding works. The corresponding corrections in the text and the bibliography are highlighted in red. The paper [DK22] is highlighted in blue because it is a preprint Arxiv-preprint version and we are not sure if it is possible to refer to it in this journal.

Answering more serious questions, we have done the following, highlighting the corrections in red.

1) We reformulated the third paragraph on p. 2 according to the new information we obtain from you.

2) We replaced $(\partial_t + L)$ by $(\partial_t - L)$ following the related tradition, though ideologically, it is better to add strongly elliptic operators than to subtract.

3) You are right, we meant that \mathcal{I} is an open interval; this is somehow 'translation difficulties' because in russian analytic tradition we usually use the word 'interval' for the corresponding open subset of the real axis and the word 'segment' for the closed one. We added the word 'open' to avoid misunderstandings.

4) As we always may proceed with a decomplexification, doubling the dimensions of vectors and matrices, but preserving the ellipticity properties of the operator L , we decided to assume all the vector functions and coefficients of differential operators under the consideration to be real-valued. Thus, the answers to the related questions on Φ^* and the function W in formula (1.5) become clear. We added the corresponding line on page 4 of the new version. Of course, in this case Φ^* and Φ^t coincides, but we still prefer to write Φ^* .

5) You are right, we need (UCP) with respect to the space variables for both the operator \mathcal{L} and \mathcal{L}^* ; we corrected the text but separated the necessity and sufficiency, i.e. the cases where (UCP) is used for the related operators.

6) You are right, $G_2 \neq (\mathbb{R}^n \times \mathcal{I})$. We are sorry, we assume that both G_1 and G_2 satisfy the assumption (A) and corrected the assumption as well.

7) Right. We added the line related to the fact that the proof is very closed to the arguments by B. Malgrange and F.E. Browder.

8) OK. We shifted the related text to a more appropriate location after formula (1.7).

9) We changed the text according to the recommendation. The problem with the complex conjugation was already discussed in 4). We also add some lines to explain why W can be extended as a distribution to $\mathbb{R}^n \times \mathcal{I}$.

10) We corrected a little bit the assumption (A) and added some arguments, why W vanishes on unbounded components of $\mathbb{R}^{n+1} \setminus \overline{G_1}$. We also thank you for the useful suggestions on the simplification of the proof. We corrected the proof according to your recommendations.

11) Probably, you are right. It seems that simple arguments with the use of the standard a priori estimates are valid for cylinder domains, only (or, more generally, for domains where the Cauchy-Dirichlet problem is well-posed). We presented a proof with the use of (UCP) for \mathcal{L} and the parametrix method for differential equations, but under additional assumptions on the smoothness of the coefficients of the elliptic operator L with respect to the time variable. For this reason we added a reference to one more book by N. Tarkhanov to the bibliography.

12) We have done the corresponding corrections in the formulation and in the proofs related to the approximation in the mean similar to that, discussed in 5), 6), 10), 11) (Theorem 2.2 in the old version and Theorems 2.2 and 2.4 in the revised one). The problem with the complex conjugation was already discussed in 4).

13) Ok. We followed the recommendation to delete two paragraphs.

14) You are right. We corrected the text.

Sincerely, Alexander Shlapunov, Pavel Vilkov.